

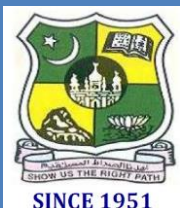
2017

# Mathmation

Compendium of Mathematics Information

Volume - 5

January - December 2017

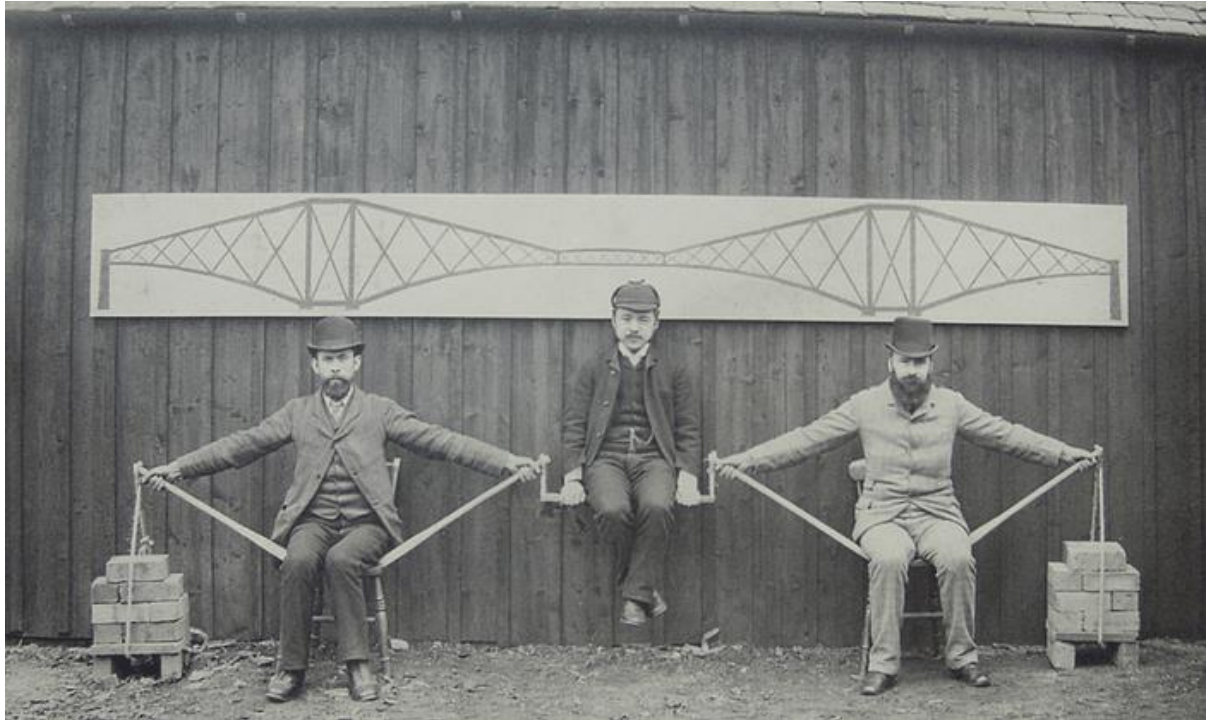


Students Endeavour  
PG & Research Department of Mathematics  
Jamal Mohamed College (Autonomous)  
College with Potential for Excellence  
Accredited with A Grade by NAAC  
(Affiliated to Bharathidasan University)  
Trichy- 20



## Introduction

This booklet is the brainchild of motivated Mathematics students & Scholars who wish to disseminate mathematical information regarding the reputed Mathematical Institutions, current events, unsolved problems, Millennium prize problems, puzzles, solutions etc.,



*A demonstration of the mathematical principles of the original Forth Bridge in Scotland performed at Imperial College in 1887. The central 'weight' is Kaichi Watanabe, one of the first Japanese engineers to study in the UK, while Sir John Fowler and Benjamin Baker provide the supports. Photograph: Imperial College*



# **JAMAL MOHAMED COLLEGE (Autonomous)**

**College with Potential for Excellence**

Reaccredited (3<sup>rd</sup> Cycle) with 'A' Grade by NAAC

(Affiliated by Bharathidasan University)

Tiruchirappalli – 620020.

## **Founders**



**Hajee M.JAMAL MOHAMED**



**Janab N.M.KHAJAMIAN ROWTHER**

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Our hearty thanks to Janab M.J. Jamal Mohamed Bilal, President, Dr. A.K. Khaja Nazemudeen, Secretary and Correspondent, Hajee M.J. Jamal Mohamed, Treasurer, Dr. K. Abdus Samad, Assistant Secretary, Dr.S. Ismail Mohideen, Principal & Head and staff members of the Department of Mathematics.

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## **PG AND RESEARCH DEPARTMENT OF MATHEMATICS**

### **VISION**

To assist students in acquiring a conceptual understanding of the nature and structure of Mathematics, its processes and applications.

### **MISSION**

To provide quality Mathematical course work which supports and enhances all programmes at the College.

### **PROFILE OF THE DEPARTMENT (2016-17)**

The Department of the Mathematics is one of the oldest departments which maintain a good record of service from its inception in 1951 the year in which the institution was established. B.Sc and M.Sc. Mathematics Programmes were started in 1957 and 1963 respectively. The department was elevated into a research department in the year 2002 by offering M.Phil (Full Time & Part Time) and Ph.D. (Full Time & Part Time) Programmes. B.Sc. Mathematics (Self Financing Section) was started for Women in 2003 and M.Sc. Mathematics (Self Financing Section) was started for Women and Men in 2005 and 2010 respectively.

In the year 2008, the department was Nationally Identified as a Department with Potential for award of FIST Grant by the Ministry of Science and Technology of Government of India.

The department feels proud of having highly qualified faculty members actively engaged in Teaching, Research, Continuing education programmes and Consultancy. In the last Twelve years, the members of the department have written 10 books, 58 research scholars were awarded Ph.D. Degrees, Published 461 research papers in National and International Journal and presented many research papers in National and International Conferences.

## **UNIQUE FEATURES**

- DST-FIST Sponsored Department (2009-2014) : Total Grants received 13 Lakhs
- 58Ph.Ds Awarded (From 2002 to 2017)
- Department Library with 8174 Books.
- CSIR / NET Coaching Class (initiated in 2016)
- VIPNET – Dr. A.P.J. Abdul Kalam Club (affiliated to VigyanPrasar Network of Science Clubs, DST-Government of India) (established in 2016)
- Ramanujan Centre for Mathematic Excellence (established in 2017)

## **THRUST AREAS OF RESEARCH**

- Algebra
- Fuzzy Graphs
- Fuzzy Groups
- Fuzzy Optimization
- Graph Theory
- Mathematical Modelling
- Network Optimization
- Operations Research
- Stochastic Process

## **SIGNIFICANT ACHIEVEMENTS**

- No. of Books Published : 10
- Papers Published in National and International Journals:461
- Presented many research papers in Conferences and Seminars
- Research Advisors- M.Phil.: 11;Ph.D.: 07
- No. of UGC Minor Research Projects completed: 6
- International Conferences organized: 3
- Regularly organizing Conferences / Seminars / Workshops
- Staff members are acting as reviewers of National / International Journals
- Our students have won many shields for overall first position in Inter-Collegiate Competitions
- Our students attended advanced learning programmes in Mathematics organized by NBHM

## BEST PRACTICES

- Publishing Mathmation Magazine (started in 2013)
- Remedial Classes by Research Scholars (Student Faculty) (started in 2016)
- Advanced Learners Forum: Students are encouraged to participate in the Seminars / Competitions by the Senior Students. (initiated in 2016)
- Organized Inter-Department Competition (Maths- Intelligentsia) (initiated in 2017)
- State Level Inter-Collegiate Meet – JAMATICS- Annual Events
- Mathematics Association organizing Special Lecture Programmes
- Our students actively participating in Co-curricular and Extra-curricular activities
- Online Competitive Examination Software
- Lectures through web NPTEL-Web and Video Courses

## ACADEMIC TRANSFORMATION

YEAR	COURSES	STREAM	SANCTIONED STRENGTH
1957	B.Sc. Mathematics	Aided	60
1963	M.Sc. Mathematics	Aided	35
2002	M.Phil. & Ph.D. Mathematics	Self Finance	As per availability of Research Advisors
2003	B.Sc. Mathematics (Addl.Sec-I)	Self Finance (Women)	60
2005	M.Sc. Mathematics (Addl.Sec-I)	Self Finance (Women)	35
2009	B.Sc. Mathematics (Addl.Sec-II)	Self Finance (Women)	60
2009	UGC Sponsored COP – E-Mathematical Tools	Self Finance	60
2010	M.Sc. Mathematics (Addl.Sec-II)	Self Finance (Men)	35
2015	B.Sc. Mathematics (Addl.Sec-III)	Self Finance (Women)	60

## Why is group theory important?

Broadly speaking, group theory is the study of symmetry. When we are dealing with an object that appears symmetric, group theory can help with the analysis. We apply the label symmetric to anything which stays invariant under some transformations. This could apply to geometric figures (a circle is highly symmetric, being invariant under any rotation), but also to more abstract objects like functions:  $x^2 + y^2 + z^2$  is invariant under any rearrangement of  $x$ ,  $y$ , and  $z$  and the trigonometric functions  $\sin(t)$  and  $\cos(t)$  are invariant when we replace  $t$  with  $t+2\pi$ .

Conservation laws of physics are related to the symmetry of physical laws under various transformations. For instance, we expect the laws of physics to be unchanging in time. This is an invariance under "translation" in time, and it leads to the conservation of energy. Physical laws also should not depend on where you are in the universe. Such invariance of physical laws under "translation" in space leads to conservation of momentum. Invariance of physical laws under

(suitable) rotations leads to conservation of angular momentum. A general theorem that explains how conservation laws of a physical system must arise from its symmetries is due to Emmy Noether.

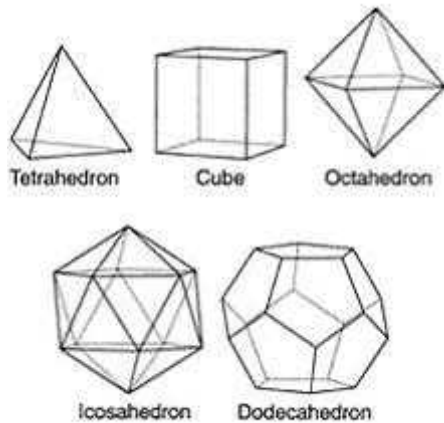
Modern particle physics would not exist without group theory; in fact, group theory predicted the existence of many elementary particles before they were found experimentally.

The structure and behavior of molecules and crystals depends on their different symmetries. Thus, group theory is an essential tool in some areas of chemistry.

Within mathematics itself, group theory is very closely linked to symmetry in geometry. In the Euclidean plane  $\mathbb{R}^2$ , the most symmetric kind of polygon is a regular polygon. We all know that for any  $n > 2$ , there is a regular polygon with  $n$  sides: the equilateral triangle for  $n = 3$ , the square for  $n = 4$ , the regular pentagon for  $n = 5$ , and so on. What are the possible regular polyhedra (like a regular pyramid and cube) in  $\mathbb{R}^3$

and, to use a more encompassing term, regular "polytopes" in  $R^d$  for  $d > 3$ ?

In  $R^3$ , there are only five (convex) regular polyhedra, called the Platonic solids:



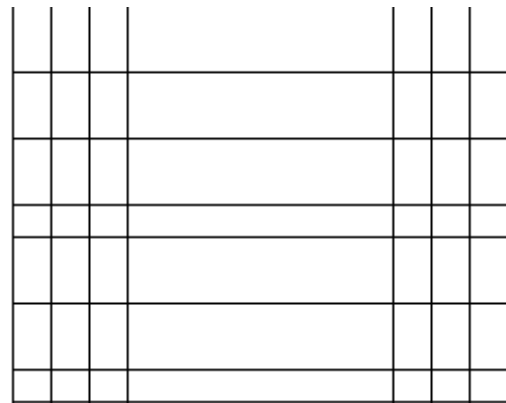
***The Five Platonic Solids***

In  $R^4$ , there are only six (convex) regular polytopes.

For any  $d > 4$ , the number of (convex) regular polytopes in  $R^d$  is always three: the higher-dimensional analogues of the tetrahedron, cube, and octahedron.

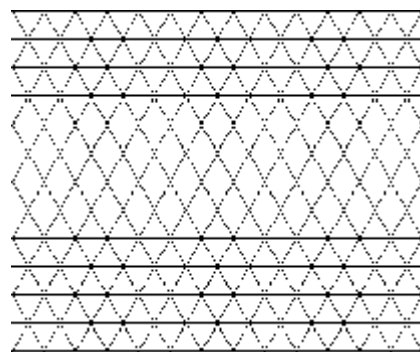
The reason there are only a few regular figures in each  $R^d$  for  $d > 2$ , but there are infinitely many regular polygons in  $R^2$ , is connected to the possible finite groups of rotations in Euclidean space of different dimensions.

Consider another geometric topic: regular tilings of the plane. This means a tiling of the plane by copies of congruent regular polygons, with no overlaps except along the boundaries of the polygons. For instance, a standard sheet of graph paper illustrates a regular tiling of  $R^2$  by squares (with 4 meeting at each vertex).



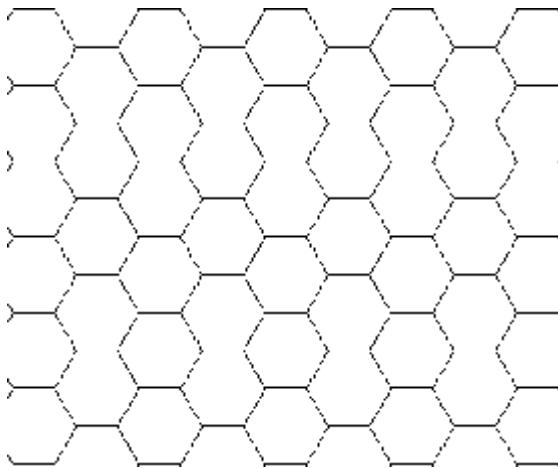
***Tiling the Plane with Congruent Squares***

There are also regular tilings of  $R^2$  by equilateral triangles (with 6 meeting at each vertex) and by regular hexagons (with 3 meeting at each vertex).



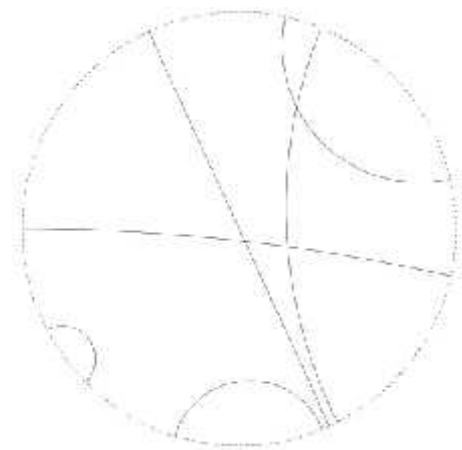


*Tiling the Plane with Congruent Equilateral Triangles*



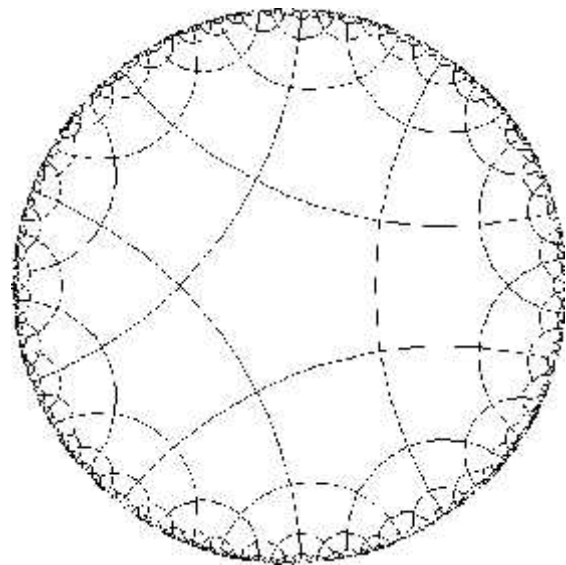
*Tiling the Plane with Congruent Regular Hexagons*

But that is all: there are no tilings of  $\mathbb{R}^2$  by (congruent) regular  $n$ -gons except when  $n = 3, 4,$  and  $6$  (e.g., how could regular pentagons meet around a point without overlap). Furthermore, for these three values of  $n$  there is essentially just one regular tiling in each case. The situation is different if we work with regular polygons in the hyperbolic plane  $\mathbb{H}^2$ , rather than the Euclidean plane  $\mathbb{R}^2$ . The hyperbolic plane  $\mathbb{H}^2$  is the interior of a disc in which the "lines" are diameters passing through the center of the disc or pieces of circular arcs inside the disc that meet the boundary of the disc in 90 degree angles. See the figure below.



*Lines in the Hyperbolic Plane  $\mathbb{H}^2$*

In  $\mathbb{H}^2$  there are tilings by (congruent) regular  $n$ -gons for any value of  $n > 2$ ! Here is a tiling of  $\mathbb{H}^2$  by congruent regular pentagons.

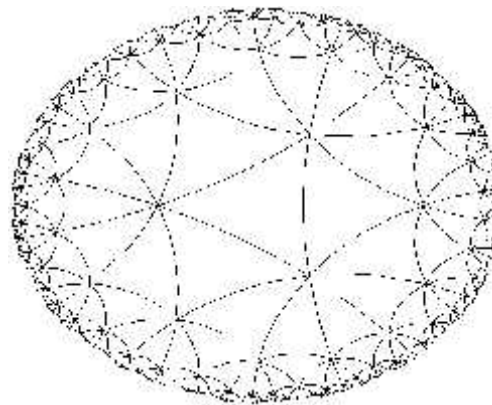


*Tiling the Hyperbolic Plane with Congruent Regular Pentagons*

The figures here are pentagons because their boundaries consists of five hyperbolic line segments (intervals along a circular arc meeting

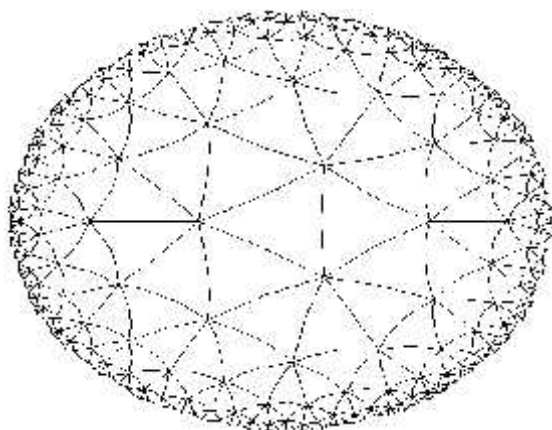
the boundary at 90 degree angles). The boundary parts of each pentagon all have the same hyperbolic length (certainly not the same Euclidean length!), so they are regular pentagons, and the pentagons are congruent to each other in  $H^2$  even though they don't appear congruent as figures in the Euclidean plane. Thus we have a tiling of  $H^2$  by congruent regular pentagons with four meeting at each vertex. Nothing like this is possible for tilings of  $R^2$ , the Euclidean plane. There is also more than one tiling of  $H^2$  by regular  $n$ -gons for the same  $n$ . For instance, taking  $n = 3$ , there is no tiling of  $H^2$  by congruent equilateral triangles meeting 6 at a vertex, but there are tilings of  $H^2$  with 7 meeting at a vertex and 8 meeting at a vertex. Here they are.

*Tiling the Hyperbolic Plane with Congruent Equilateral Triangles, 7 at a Vertex*



*Tiling the Hyperbolic Plane with Congruent Equilateral Triangles, 8 at a Vertex*

What makes it possible to tile  $H^2$  by regular polygons in more possible ways than  $R^2$  is the different structure of the group of rigid motions (distance-preserving transformations) in  $H^2$  compared to  $R^2$ . The German mathematician Felix Klein gave a famous lecture in Erlangen, in which he asserted that the definition of a geometry is the study of the properties of a space that are invariant under a chosen group of transformations of that space.



Group theory shows up in many other areas of geometry. For instance, in addition to attaching numerical invariants to a space (such as its dimension, which is just a

number) there is the possibility of introducing algebraic invariants of a space. That is, one can attach to a space certain algebraic systems. Examples include different kinds of groups, such as the fundamental group of a space. A plane with one point removed has a commutative fundamental group, while a plane with two points removed has a noncommutative fundamental group. In higher dimensions, where we can't directly visualize spaces that are of interest, mathematicians often rely on algebraic invariants like the fundamental group to help us verify that two spaces are not the same.

Classical problems in algebra have been resolved with group theory. In the Renaissance, mathematicians found analogues of the quadratic formula for roots of general polynomials of degree 3 and 4. Like the quadratic formula, the cubic and quartic formulas express the roots of all polynomials of degree 3 and 4 in terms of the coefficients of the polynomials and root extractions (square roots, cube roots, and fourth roots). The search for an analogue of

the quadratic formula for the roots of all polynomials of degree 5 or higher was unsuccessful. In the 19th century, the reason for the failure to find such general formulas was explained by a subtle algebraic symmetry in the roots of a polynomial discovered by Evariste Galois. He found a way to attach a finite group to each polynomial  $f(x)$ , and there is an analogue of the quadratic formula for all the roots of  $f(x)$  exactly when the group associated to  $f(x)$  satisfies a certain technical condition that is too complicated to explain here. Not all groups satisfy the technical condition, and by this method Galois could give explicit examples of fifth degree polynomials, such as  $x^5 - x - 1$ , whose roots can't be described by anything like the quadratic formula. Learning about this application of group theory to formulas for roots of polynomials would be a suitable subject for a second course in abstract algebra.

The mathematics of public-key cryptography uses a lot of group theory. Different cryptosystems use different groups, such as the group of units in modular arithmetic and the

group of rational points on elliptic curves over a finite field. This use of group theory derives not from the "symmetry" perspective, but from the efficiency or difficulty of carrying out certain computations in the groups. Other public-key cryptosystems use other algebraic structures, such as lattices.

Some areas of analysis (the mathematical developments coming from calculus) involve group theory. The subject of Fourier series is concerned with expanding a fairly general  $2\pi$ -periodic function as an infinite series in the special  $2\pi$ -periodic functions  $1$ ,  $\sin(x)$ ,  $\cos(x)$ ,  $\sin(2x)$ ,  $\cos(2x)$ ,  $\sin(3x)$ ,  $\cos(3x)$ , and so on. While it can be developed solely as a topic within analysis (and at first it was), the modern viewpoint of Fourier analysis treats it as a fusion of analysis, linear algebra, and group theory.

Identification numbers are all around us, such as the ISBN number for a book, the VIN (Vehicle Identification Number) for your car, or the bar code on a UPS package. What makes them useful is their check digit,

which helps catch errors when communicating the identification number over the phone or the internet or with a scanner. The different recipes for constructing a check digit from another string of numbers are based on group theory. Usually the group theory is trivial, just addition or multiplication in modular arithmetic. However, a more clever use of other groups leads to a check-digit construction which catches more of the most common types of communication errors. The key idea is to use a noncommutative group.

On the lighter side, there are applications of group theory to puzzles, such as the 15-puzzle and Rubik's Cube. Group theory provides the conceptual framework for solving such puzzles. To be fair, you can learn an algorithm for solving Rubik's cube without knowing group theory (consider this 7-year old cubist), just as you can learn how to drive a car without knowing automotive mechanics. Of course, if you want to understand how a car works then you need to know what is really going on under the hood. Group theory

(symmetric groups, conjugations, commutators, and semi-direct

products) is what you find under the hood of Rubik's cube.

### Quasi-crystals and the Golden Ratio

**Quasi-crystals represent a newly discovered state of matter.**

Most crystals in nature, such as those in sugar, salt or diamonds, are symmetrical and all have the same orientation throughout the entire crystal. Quasicrystals represent a new state of matter that was not expected to be found, with some properties of crystals and others of non-crystalline matter, such as glass.

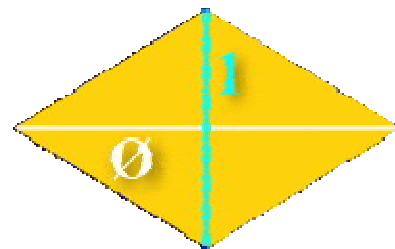
With five-fold symmetry, once thought to be impossible, they were first observed in 1982 in an aluminium-manganese alloy ( $Al_6Mn$ ). Since then, quasicrystals have been found in other substances.

**Quasi-crystals fill space with five-fold symmetry based on phi.**

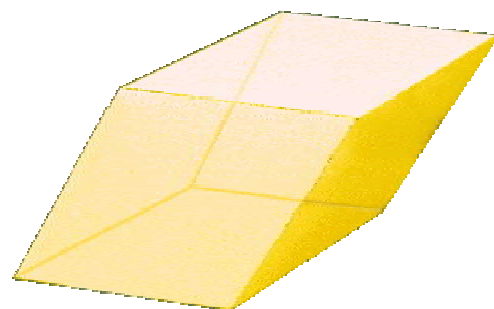
Penrose tiles allow a two-dimensional area to be filled in five-fold symmetry, using two shapes based on phi. It was thought that filling a three-dimensional space in

five-fold symmetry was impossible, but the answer was again found in phi.

Where the solution in 2D required two shapes, this can be accomplished in 3D with just one shape. The shape has six sides, each one a diamond whose diagonals are in the ratio of phi:



The resulting solid looks like this:



A Nobel Prize for Quasi-Crystal Discovery was awarded in 2011.

As reported in "2011 Nobel Prize in Chemistry: 'Quasicrystals' once thought impossible have changed

understanding of solid matter” by ScienceDaily on October 14, 2011.

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Chemistry for 2011 to Daniel Shechtman of the Technion – Israel Institute of Technology in Haifa, Israel, for the discovery of quasicrystals: non-repeating regular patterns of atoms that were once thought to be impossible.

In quasicrystals, we find the fascinating mosaics of the Arabic world reproduced at the level of atoms: regular patterns that never repeat themselves. However, the configuration found in quasicrystals was considered impossible, and Daniel Shechtman had to fight a fierce battle against established science. The Nobel Prize in Chemistry 2011 recognizes a breakthrough that has fundamentally altered how chemists conceive of solid matter.

On the morning of April 8, 1982, an image counter to the laws of nature appeared in Daniel Shechtman’s electron microscope. In all solid matter, atoms were believed

to be packed inside crystals in symmetrical patterns that were repeated periodically over and over again. For scientists, this repetition was required in order to obtain a crystal.

Shechtman’s image, however, showed that the atoms in his crystal were packed in a pattern that could not be repeated. Such a pattern was considered just as impossible as creating a football using only six-cornered polygons, when a sphere needs both five- and six-cornered polygons. His discovery was extremely controversial. In the course of defending his findings, he was asked to leave his research group. However, his battle eventually forced scientists to reconsider their conception of the very nature of matter.

Aperiodic mosaics, such as those found in the medieval Islamic mosaics of the Alhambra Palace in Spain and the Darb-i Imam Shrine in Iran, have helped scientists understand what quasicrystals look like at the atomic level. In those

mosaics, as in quasicrystals, the patterns are regular – they follow mathematical rules – but they never repeat themselves.

When scientists describe Shechtman's quasicrystals, they use a concept that comes from mathematics and art: the golden ratio. This number had already caught the interest of mathematicians in Ancient Greece, as it often appeared in geometry. In quasicrystals, for instance, the ratio of various distances between atoms is related to the golden mean.

Following Shechtman's discovery, scientists have produced other kinds of quasicrystals in the lab and discovered naturally occurring quasicrystals in mineral samples from a Russian river. A Swedish company has also found quasicrystals in a certain form of steel, where the crystals reinforce the material like armor. Scientists are currently experimenting with using quasicrystals in different products such as frying pans and diesel engines.

### **History of the findings**

These findings came about as follows:

In the mid-1970s the mathematician Roger Penrose created an aperiodic mosaic, with a pattern that never repeats itself, using only two different tiles: one fat rhomboid and one thin rhomboid. He called these kites and darts and named this finding Penrose tiles.

In 1982, Dan Shechtman captured a picture with an electronic microscope that seemed counter to all logic. The ten bright dots in each circle revealed that he was seeing tenfold symmetry. Conventional wisdom said that this was against the laws of nature.

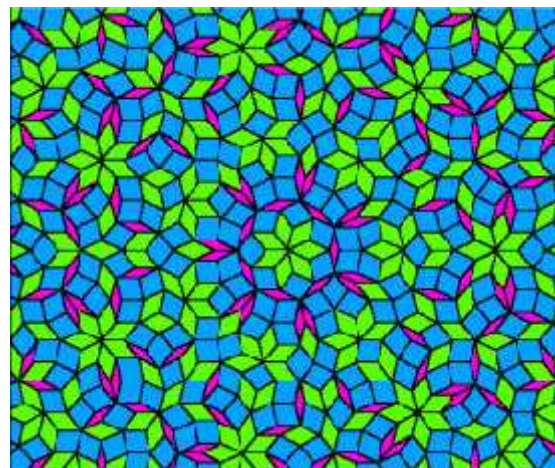
In 1982, Alan Mackay experimented with a model in which circles representing atoms were placed at intersections in Penrose's mosaic. He illuminated the model and found a tenfold diffraction pattern.

In 1984, Paul Steinhardt and Dov Levine connected Mackay's model with Shechtman's actual diffraction pattern. They realized that

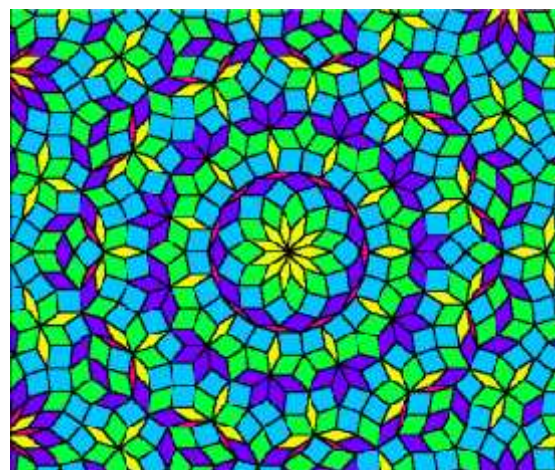
aperiodic mosaics helped to explain Shechtman’s unusual crystals.

**Implications for other geometries**

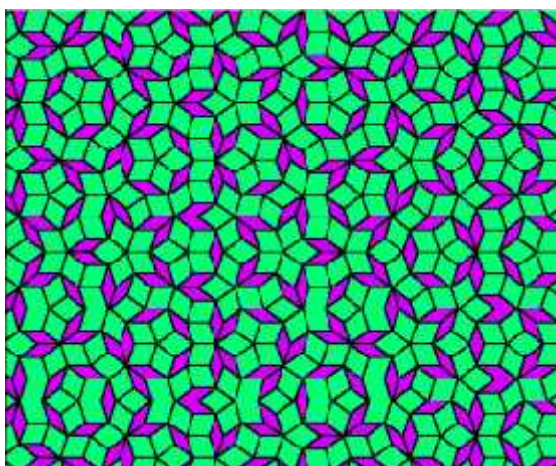
One of the most amazing implications of this property is that it’s not just 5-fold symmetry that is made possible. The unexpected find is that with quasiperiodicity, a whole new class of solids is possible! Any symmetry in any number of dimensions becomes attainable! Here are some examples of other symmetries from a presentation by P.J. Steinhardt titled “What are quasicrystals”



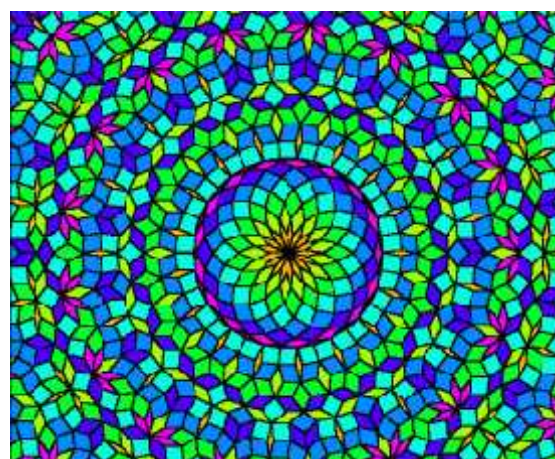
*Quasi-periodicity 7 fold symmetry*



*Quasi-periodicity 11 fold symmetry*



*Quasi-periodicity 5 fold symmetry*



*Quasi-periodicity 17 fold symmetry*



**References:**

[https://www.nobelprize.org/nobel\\_prizes/chemistry/laureates/2011/popular-chemistryprize2011.pdf](https://www.nobelprize.org/nobel_prizes/chemistry/laureates/2011/popular-chemistryprize2011.pdf)

[http://www.isis.org.uk/Golden\\_Mean\\_Wins\\_Chemistry\\_Nobe\\_Prize.php](http://www.isis.org.uk/Golden_Mean_Wins_Chemistry_Nobe_Prize.php)

<http://www.physics.princeton.edu/~steinh/QuasiIntro.ppt>

**Fibonacci 60 Repeating Pattern**

**Fibonacci** (c. 1175 – c. 1250) was an Italian mathematician from the Republic of Pisa, considered to be "the most talented Western mathematician of the Middle Ages". The name he is commonly called, "Fibonacci" (Italian: [fibo natti]), was made up in 1838 by the French historian Guillaume Libri and is short for "filiusBonacci" ("son of (the) Bonacci") and he is also known as **Leonardo Bonacci, Leonardo of Pisa, Leonardo Pisano Bigollo, or Leonardo Fibonacci.**

Fibonacci popularized the Hindu-Arabic numeral system in the Western World primarily through his composition in 1202 of *Liber Abaci* (*Book of Calculation*). He also introduced Europe to the sequence

of Fibonacci numbers, which he used as an example in *Liber Abaci*.

In mathematics, the Fibonacci numbers are the numbers in the following integer sequence, called the Fibonacci sequence, and characterized by the fact that every number after the first two is the sum of the two preceding ones:

1,1,2,3,5,8,13,21,34,55,89,144,...

Often, especially in modern usage, the sequence is extended by one more initial term:

1,1,2,3,5,8,13,21,34,55,89,144,...

By definition, the first two numbers in the Fibonacci sequence are either 1 and 1, or 0 and 1, depending on the chosen starting point of the sequence, and each subsequent

number is the sum of the previous two.

The sequence  $F_n$  of Fibonacci numbers is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2}$$

With seed values

$$F_1 = 1, \quad F_2 = 1$$

Or

$$F_0 = 0, \quad F_1 = 1$$

**Fibonacci 60 Repeating Pattern**

The last digit of the numbers in the Fibonacci Sequence form a pattern that repeats after every 60th number:

0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, 1, 7, 8, 5, 3, 8, 1, 9, 0, 9, 9, 8, 7, 5, 2, 7, 9, 6, 5, 1, 6, 7, 3, 0, 3, 3, 6, 9, 5, 4, 9, 3, 2, 5, 7, 2, 9, 1

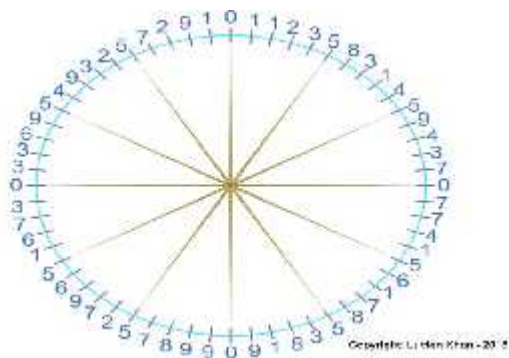
This pattern can be seen in the following list of the first 72 Fibonacci numbers:

0	<b>0</b>
1	<b>1</b>
2	<b>1</b>
3	<b>2</b>
4	<b>3</b>
5	<b>5</b>
6	<b>8</b>
7	<b>13</b>
8	<b>21</b>
9	<b>34</b>
10	<b>55</b>
11	<b>89</b>
12	<b>144</b>
13	<b>233</b>
14	<b>377</b>
15	<b>610</b>

16	<b>987</b>
17	<b>1597</b>
18	<b>2584</b>
19	<b>4181</b>
20	<b>6765</b>
21	<b>10946</b>
22	<b>17711</b>
23	<b>28657</b>
24	<b>46368</b>
25	<b>75025</b>
26	<b>121393</b>
27	<b>196418</b>
28	<b>317811</b>
29	<b>514229</b>
30	<b>832040</b>
31	<b>1346269</b>
32	<b>2178309</b>
33	<b>3524578</b>
34	<b>5702887</b>
35	<b>9227465</b>
36	<b>14930352</b>
37	<b>24157817</b>
38	<b>39088169</b>
39	<b>63245986</b>
40	<b>102334155</b>
41	<b>165580141</b>
42	<b>267914296</b>
43	<b>433494437</b>
44	<b>701408733</b>
45	<b>1134903170</b>
46	<b>1836311903</b>
47	<b>2971215073</b>
48	<b>4807526976</b>
49	<b>7778742049</b>
50	<b>12586269025</b>
51	<b>20365011074</b>
52	<b>32951280099</b>
53	<b>53316291173</b>
54	<b>86267571272</b>
55	<b>139583862445</b>
56	<b>225851433717</b>
57	<b>365435296162</b>
58	<b>591286729879</b>
59	<b>956722026041</b>
60	<b>1548008755920</b>
61	<b>2504730781961</b>
62	<b>4052739537881</b>
63	<b>6557470319842</b>

64	<b>10610209857723</b>
65	<b>17167680177565</b>
66	<b>27777890035288</b>
67	<b>44945570212853</b>
68	<b>72723460248141</b>
69	<b>117669030460994</b>
70	<b>190392490709135</b>
71	<b>3080611521170129</b>
72	<b>498454011879264</b>

Lucien Khan arranged these 60 digits of the pattern in a circle, as shown in illustration below



Here he found other interesting results:

- The zeros align with the 4 cardinal points on a compass.
- The fives align with the 8 other points of the 12 points on a clock.
- Except for the zeros, the number directly opposite each number adds to 10.

Lucien postulates that ancient knowledge of these relationships contributed to the development of our modern use of 60 minutes in an hour,

and presentation of numbers on the face of the clock.

Any group of four numbers that are 90 degrees from each other (15 away from each other in the circle) sum to 20, except again for the zeros. As an example, use 1, 7, 9 and 3, which appear one to the right of each of the compass points.

Additionally, every group of five numbers that define the points of the 12 pentagons on the circle also create a pattern. Four of the pentagons have even-numbered last digits of 0, 2, 4, 6, and 8. The remaining eight pentagons have odd-numbered last digits of 1, 3, 5, 7 and 9.

Points of each Pentagon on the Circle (separated by 12 points):

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60

Last digit of the Fibonacci Sequence number at that Point:

0	1	1	2	3	5	8	3	1	4	5	4
1	3	7	0	7	7	1	1	5	6	1	7
3	5	3	3	1	5	0	9	5	3	7	2
2	7	0	5	5	1	5	7	1	0	3	3
5	0	5	4	0	3	2	5	7	2	0	1

Another interesting pattern yet was observed by Lucien Khan: The 216th number in this sequence is 619220451666590135228675387863297874269396512. The sum of all the digits in that number add up to 216, as well.

He notes that it is believed that the secret or hidden name of God contains 216 characters. There are many other fascinating relationships and sacred geometries, which are presented by Lucien Khan in more detail at the links below.

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<https://docs.google.com/document/d/1mVWd1aLiYZQU8VvYFBnW8kxodeYim3bYDIFfh-w42eU/pub>

## The 10 Most Beautiful Mathematical Equations

Courtesy Livescience

### Introduction

Mathematical equations aren't just useful – many are quite beautiful. And many scientists admit they are often fond of particular formulas not just for their function, but for their form, and the simple, poetic truths they contain.

While certain famous equations, such as Albert Einstein's  $E = mc^2$ , hog most of the public glory, many less familiar formulas have their champions among scientists.

### 1. General Relativity

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \rho_- \wedge g_{\mu\nu})$$

The equation above was formulated by Einstein as part of his groundbreaking general theory of relativity in 1915. The theory revolutionized how scientists

understood gravity by describing the force as a warping of the fabric of space and time.

"It is still amazing to me that one such mathematical equation can describe what space-time is all about," said Space Telescope Science Institute astrophysicist Mario Livio, who nominated the equation as his favorite. "All of Einstein's true genius is embodied in this equation."

"The right-hand side of this equation describes the energy contents of our universe (including the 'dark energy' that propels the current cosmic acceleration)," Livio explained. "The left-hand side describes the geometry of space-time. The equality reflects the fact that in Einstein's general relativity, mass and energy determine the geometry, and concomitantly the

curvature, which is a manifestation of what we call gravity."

"It's a very elegant equation," said Kyle Cranmer, a physicist at New York University, adding that the equation reveals the relationship between space-time and matter and energy. "This equation tells you how they are related – how the presence of the sun warps space-time so that the Earth moves around it in orbit, etc. It also tells you how the universe evolved since the Big Bang and predicts that there should be black holes."

## 2. Calculus

$$\int_a^b f'(x) dx = f(b) - f(a)$$

While the first two equations describe particular aspects of our universe, another favorite equation can be applied to all manner of situations. The fundamental theorem of calculus forms the backbone of the mathematical method known as calculus, and links its two main ideas, the concept of the integral and the concept of the derivative.

"In simple words, [it] says that the net change of a smooth and continuous quantity, such as a distance travelled, over a given time interval (i.e. the difference in the values of the quantity at the end points of the time interval) is equal to the integral of the rate of change of that quantity, i.e. the integral of the velocity," said Melkana Brakalova-Trevithick, chair of the math department at Fordham University, who chose this equation as her favorite. "The fundamental theorem of calculus (FTC) allows us to determine the net change over an interval based on the rate of change over the entire interval."

The seeds of calculus began in ancient times, but much of it was put together in the 17th century by Isaac Newton, who used calculus to describe the motions of the planets around the sun.

## 3. Pythagorean theorem

$$a^2 + b^2 = c^2$$

An "oldie but goodie" equation is the famous Pythagorean theorem,

which every beginning geometry student learns.

This formula describes how, for any right-angled triangle, the square of the length of the hypotenuse,  $c$ , (the longest side of a right triangle) equals the sum of the squares of the lengths of the other two sides ( $a$  and  $b$ ). Thus,

$$a^2 + b^2 = c^2$$

"The very first mathematical fact that amazed me was Pythagorean theorem," said mathematician Daina Taimina of Cornell University. "I was a child then and it seemed to me so amazing that it works in geometry and it works with numbers!"

**4.  $1 = 0.999999999\dots$**

$$1 = 0.9999999999999999 \dots$$

This simple equation, which states that the quantity 0.999, followed by an infinite string of nines, is equivalent to one, is the favorite of mathematician Steven Strogatz of Cornell University.

"I love how simple it is – everyone understands what it says – yet how provocative it is," Strogatz said. "Many people don't believe it

could be true. It's also beautifully balanced. The left side represents the beginning of mathematics; the right side represents the mysteries of infinity."

**5. Special Relativity**

$$t' = t \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Einstein makes the list again with his formulas for special relativity, which describes how time and space aren't absolute concepts, but rather are relative depending on the speed of the observer. The equation above shows how time dilates, or slows down, the faster a person is moving in any direction.

"The point is it's really very simple," said Bill Murray, a particle physicist at the CERN laboratory in Geneva. "There is nothing there an A-level student cannot do, no complex derivatives and trace algebras. But what it embodies is a whole new way of looking at the world, a whole attitude to reality and our relationship to it. Suddenly, the rigid unchanging cosmos is swept away and replaced

with a personal world, related to what you observe. You move from being outside the universe, looking down, to one of the components inside it. But the concepts and the maths can be grasped by anyone that wants to."

Murray said he preferred the special relativity equations to the more complicated formulas in Einstein's later theory. "I could never follow the maths of general relativity," he said.

### 6. Euler's equation

$$V - E + F = 2$$

This simple formula encapsulates something pure about the nature of spheres:

"It says that if you cut the surface of a sphere up into faces, edges and vertices, and let F be the number of faces, E the number of edges and V the number of vertices, you will always get  $V - E + F = 2$ ," said Colin Adams, a mathematician at Williams College in Massachusetts.

"So, for example, take a tetrahedron, consisting of four triangles, six edges and four vertices," Adams explained. "If you blew hard

into a tetrahedron with flexible faces, you could round it off into a sphere, so in that sense, a sphere can be cut into four faces, six edges and four vertices. And we see that  $V - E + F = 2$ . Same holds for a pyramid with five faces – four triangular, and one square – eight edges and five vertices," and any other combination of faces, edges and vertices.

"A very cool fact! The combinatorics of the vertices, edges and faces is capturing something very fundamental about the shape of a sphere," Adams said.

### 7. Euler-Lagrange equations and Noether's theorem

$$\frac{d}{dt} \left( \frac{\partial L}{\partial q} \right) = \frac{\partial L}{\partial q}$$

"These are pretty abstract, but amazingly powerful," NYU's Cranmer said. "The cool thing is that this way of thinking about physics has survived some major revolutions in physics, like quantum mechanics, relativity, etc."

Here, L stands for the Lagrangian, which is a measure of energy in a physical system, such as springs, or levers or fundamental

particles. "Solving this equation tells you how the system will evolve with time," Cranmer said.

A spinoff of the Lagrangian equation is called Noether's theorem, after the 20th century German mathematician Emmy Noether. "This theorem is really fundamental to physics and the role of symmetry," Cranmer said. "Informally, the theorem is that if your system has a symmetry, then there is a corresponding conservation law. For example, the idea that the fundamental laws of physics are the same today as tomorrow (time symmetry) implies that energy is conserved. The idea that the laws of physics are the same here as they are in outer space implies that momentum is conserved. Symmetry is perhaps the driving concept in fundamental physics, primarily due to [Noether's] contribution."

### 8. The Callan-Symanzik equation

$$\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + n\gamma \right] G^n(x_1, x_2, \dots, x_n; M, g) = 0$$

"The Callan-Symanzik equation is a vital first-principles equation from 1970, essential for describing how naive expectations will fail in a quantum world," said theoretical physicist Matt Strassler of Rutgers University.

The equation has numerous applications, including allowing physicists to estimate the mass and size of the proton and neutron, which make up the nuclei of atoms.

Basic physics tells us that the gravitational force, and the electrical force, between two objects is proportional to the inverse of the distance between them squared. On a simple level, the same is true for the strong nuclear force that binds protons and neutrons together to form the nuclei of atoms, and that binds quarks together to form protons and neutrons. However, tiny quantum fluctuations can slightly alter a force's dependence on distance, which has dramatic consequences for the strong nuclear force.

"It prevents this force from decreasing at long distances, and



causes it to trap quarks and to combine them to form the protons and neutrons of our world," Strassler said. "What the Callan-Symanzik equation does is relate this dramatic and difficult-to-calculate effect, important when [the distance] is roughly the size of a proton, to more subtle but easier-to-calculate effects that can be measured when [the distance] is much smaller than a proton."

### 9. The minimal surface equation

$$A(u) = \int_{\Omega} (1 + |\nabla u|^2)^{1/2} dx_1 \dots dx_n,$$

"The minimal surface equation somehow encodes the beautiful soap films that form on wire boundaries when you dip them in soapy water," said mathematician Frank Morgan of Williams College. "The fact that the equation is 'nonlinear,' involving powers and products of derivatives, is the coded mathematical hint for the surprising behavior of soap films. This is in contrast with more familiar linear partial differential equations, such as the heat equation, the wave equation,

and the Schrödinger equation of quantum physics."

### 10. The Euler line

Glen Whitney, founder of the Museum of Math in New York, chose another geometrical theorem, this one having to do with the Euler line, named after 18th-century Swiss mathematician and physicist Leonhard Euler.

"Start with any triangle," Whitney explained. "Draw the smallest circle that contains the triangle and find its center. Find the center of mass of the triangle – the point where the triangle, if cut out of a piece of paper, would balance on a pin. Draw the three altitudes of the triangle (the lines from each corner perpendicular to the opposite side), and find the point where they all meet. The theorem is that all three of the points you just found always lie on a single straight line, called the 'Euler line' of the triangle."

Whitney said the theorem encapsulates the beauty and power of mathematics, which often reveals

surprising patterns in simple, familiar shapes.



## Mathematics in Construction

### Introduction:

In the real world of building construction there are many rich problems which can be used to build sense making and reasoning skills for students.

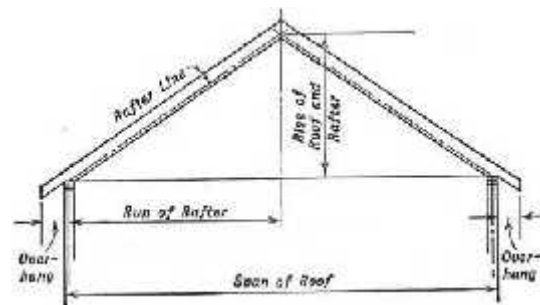
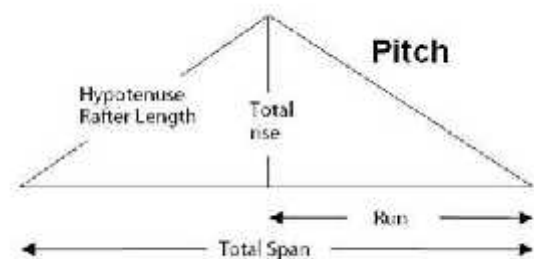
The Pythagorean Theorem is used extensively in designing and building structures, especially roofs. Gable roofs, for example, are made by placing two right triangles together. Specialized terms help to explain the triangle relationships in roof construction.

### Terminology

The span is the length from the outside wall to the outside wall of a building. Because construction is often made up of multiple layers of wood,

building plans often provided detailed descriptions to make clear where to begin or end measurements.

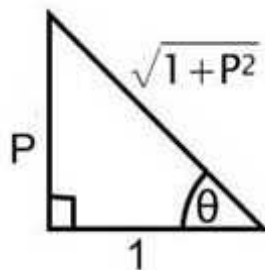
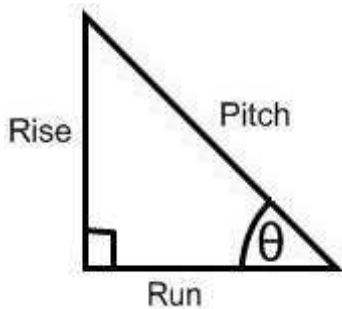
The RUN is one-half the span. Units of run are typically based on 12 inches.



Carpenters do not refer to the angle of a roof as 300 or 600, but prefer to use the pitch of the roof.

The pitch is a ratio of vertical to horizontal measurements.

If a plan calls for a 6/12 pitch roof, then the architect wants the slope of the roof to go up six inches for every 12 inches of horizontal run. Carpenters prefer to use Pitch in calculations instead of rise and run.



**Key Formulas:**

$$\text{Length of rafter} = \sqrt{(\text{rise})^2 + (\text{run})^2}$$

$$\tan\theta = \frac{\text{rise}}{\text{run}} = \text{pitch} = \frac{p}{1}$$

Suppose a roof has a rise of 18 inches and a run of 15 feet. How long is the rafter

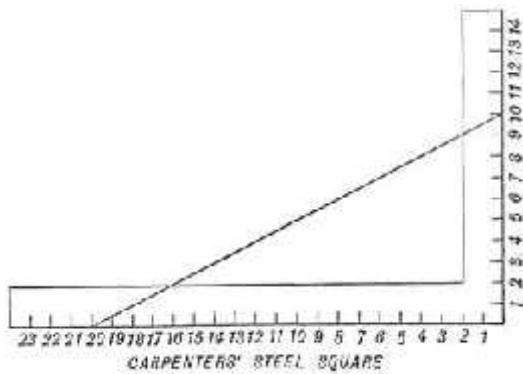
$$\begin{aligned} \text{Length of rafter} &= \sqrt{(1.5')^2 + (15)^2} \\ &= 15.075 \end{aligned}$$

Only very simple tools are needed to cut r roof rafters such as a saw, framing square (or carpenters square) and a tape measure.

The framing square is made up of two legs joined at the heel to form a right angle. The longer leg of the framing square is called the body and the shorter one leg is called the tongue. To measure a 10/20 roof pitch, place the 10 on the tongue and the 20 on the body. When a piece of lumber is added to a carpenter's square, it becomes a right triangle.



**carpenter's square**



**Math Application**

A Gable is a triangle formed by a sloping roof. A building may be front-gabled or side-gabled.

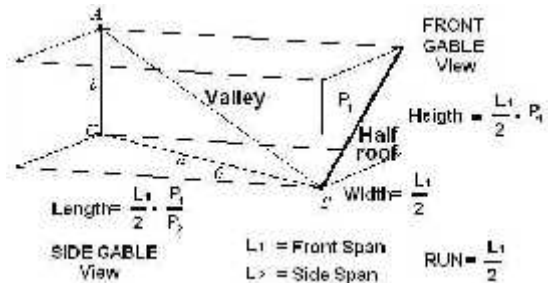
Consider structures where two roofs intersect as shown here.



When two roofs intersect, a valley rafter is formed.



The valley formed from the two common rafters and can be thought of as the diagonal in a rectangular solid.



From geometry we know that right triangles are similar therefore the sides of one triangle is proportional to the sides of the other. When the Pitch of the two intersecting roofs are not the same, see Gable picture, then the valley rafter calculation will involve a proportion.

$$Length = \frac{span}{2} \cdot \frac{Pitch1}{Pitch2}$$

Other formulas needed to work with roofs are as follows:

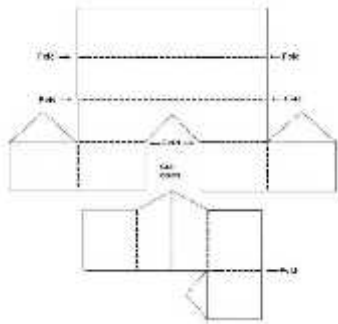
$$\begin{aligned} \tan\theta &= \frac{H}{0.5L} & H &= 0.5L \tan\theta & \text{Slope Factor} \\ & & H &= 0.5LP & \\ &= \sqrt{p^2 + 1} & &= \sqrt{\tan^2(\text{pitch angle}) + 1} \end{aligned}$$

Find the length of the valley rafter given the following measurements.

$L_1=8$  feet,  $L_2=10$  feet, Pitch 1 = 1, Pitch 2 = 0.8

$$\alpha = \sqrt{4^2 + \left(4 \cdot \frac{1}{0.8}\right)^2} = \sqrt{281} = 16.76$$

Length of Valley rafter =  $\alpha =$   
 $\sqrt{4^2 + (16.76)^2} = 17.23$



Paper version of intersecting roofs and corresponding valley rafter

**Reference:**

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### Application of Graph theory in Everyday Life

KiranKaundal

Graph theory is a branch of discrete mathematics. Graph theory is the study of graphs which are mathematical structures used to model pair wise relations between objects. A graph is made up of vertices V (nodes) and edges E (lines) that connect them. A graph is an ordered pair  $G = (V, E)$  consisting a set of vertices V with a set of edges E. Graph theory is originated with the problem of Koinsber bridge, in 1735.

This problem escort to the concept of Eulerian Graph. Euler studied the problem of Koinsberg Bridge and established a structure to

resolve the problem called Eulerian graph. In 1840, A.F Mobius presented the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles was enacted by Gustav Kirchhoff in 1845, and he enrolled graph theoretical ideas in the calculation of currents in electrical networks or circuits.

In 1852, Thomas Gutherie established the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton

measured cycles on polyhedra and contrived the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeney mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This is considered as origin of Graph Theory.

### **Graph Theory in Everyday Life**

There is n-number of applications of graph theory, few are represented as follows:

#### ***GPS or Google Maps***

GPS or Google Maps are to find a shortest route from one destination to another. The destinations are Vertices and their connections are Edges consisting distance. The optimal route is determined by the software. Schools/ Colleges are also using this technique to pick up students from their stop to school. Each stop is a vertex and the route is an edge. A Hamiltonian path represents the

efficiency of including every vertex in the route.

#### ***Traffic lights***

The functioning of traffic lights i.e. turning Green/Red and timing between them. Here vertex coloring technique is utilized to solve conflicts of time and space by identifying the chromatic number for the number of cycles needed.

#### ***Social Networks***

We connect with friends via social media or a video gets viral, here user is a Vertex and other connected users create an edge therefore videos get viral when reached to certain connections.

#### ***To clear road blockage***

When roads of a city are blocked due to ice. Planning is needed to put salt on the roads. Then Euler paths or circuits are used to traverse the streets in the most efficient way. 2.5. While using Google to search for WebPages, Pages are linked to each other by hyperlinks. Each page is a vertex and the link between two pages is an edge.

***The matching problem:***

In order to assign jobs to employees (servers) there is an analogue in software to maximize the efficiency.

**Graph Theory in Technology****Graphs in Computer Science*****Data Mining:***

Data mining is process of perceiving required information from huge data with the help of various methods. Mostly the data we deal with in data science can be shaped as graphs. These graphs can be mined utilizing known algorithms and various techniques in graph theory to understand them in better way, e.g. in social networks every person in the network could be supposed as a vertex and any connection between them is supposed as an edge. Any problem related to logistics could be modelled as a network. Graph is captivating model of data backed with a strong theory and a set of quality algorithms to solve related problems.

***GSM Mobile Phone Networks and Map Coloring:***

All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four distinct frequency ranges, it is clear by the concept of graph theory that only four colors may be utilized to color the cellular regions. These four different colors are used for proper coloring of the regions. The vertex coloring algorithm can be used to allocate at most four distinct frequencies for any GSM mobile phone network.

***Web Designing***

Website designing can be modeled as a graph, where the web pages are entitled by vertices and the hyper links between them are entitled by edges in the graph. This concept is called as web graph. Which investigate the interesting information? Other implementation areas of graphs are in web community. Where the vertices represent classes of objects, and each vertex representing one type of objects, and each vertex is connected to every vertex representing other kind of objects. In graph theory such a graph is called a complete bipartite graph.

There are many benefits graph theory in website development like: Searching and community discovery, Directed Graph is used in web site utility evaluation and link structure. Also searching all connected component and providing easy detection.

### **Graphs in OR**

Graph theory is dynamic tool in combinatorial operations research. Some important Operation Research problems which can be explained using graphs are given here. Transport network is used to model the transportation of commodity from one destination to another destination. The objective is to maximize the flow or minimize the cost within the suggested flow. The graph theory is established as more competent for these types of problems though they have more constraints.

### **Graphs in Chemistry**

The structural formulae of covalently bonded compounds are graphs; they are known as constitutional graphs. Graph theory provides the basis for definition, enumeration, systematization, codification, nomenclature, correlation, and computer programming [7]. The chemical information is associated with structural formulae and that structural formulae may be systematically and uniquely indexed and redeemed. One does translate chemical structures into words by nomenclature rules. The importance of graph theory for chemistry originates mainly from the existence of the phenomenon of isomerism, which is extenuated by chemical structure theory. This theory accounts for all constitutional isomers by using purely graph-theoretical methods.

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## 20 Excellent YouTube Channels for Math Teachers

Below is a collection featuring 20 of the best and most popular YouTube channels for math teachers. These channels provide a wide variety of videos, tutorials and animated courses covering different mathematical concepts from Algebra to Geometry.

[www.tdsb.edu/teachers](http://www.tdsb.edu/teachers)

 <p><b>Math TV</b> Khan Academy's 2017 Teacher Pick award winner. Khan Academy's math content is available on YouTube for free.</p>	 <p><b>Khan Academy</b> The world's largest free online learning platform. It offers a wide range of math courses and resources.</p>
 <p><b>Numberphile</b> This is a fun place for math. It's a place where you can learn about numbers and math in a fun and engaging way.</p>	 <p><b>Techmath</b> Techmath is a place where you can learn about math in a fun and engaging way. It's a place where you can learn about math in a way that is both fun and educational.</p>
 <p><b>PatrickJMT</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>PreProBob</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>Mathantics</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>Professor Leonard</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>Krista King</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>Free Math Videos</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>PBS Math Club</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>MashUp Math</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>Math Meeting</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>Learn math Tutorials</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>The Video Math Tutor</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>TeacherTube Math</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>Math Bff</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>Simon Deacon</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>
 <p><b>NumberRock Math Songs</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>	 <p><b>HEGARTY Maths</b> This channel is a great resource for math teachers. It provides a wide range of math lessons and resources that are easy to understand and use in the classroom.</p>

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## Top 10 Universities for Mathematics in the World Based on the QS World University Rankings by Subject 2017

### 1. Massachusetts Institute of Technology (MIT) USA



The Massachusetts Institute of Technology (MIT) is a private research university in Cambridge, Massachusetts. Founded in 1861 in response to the increasing industrialization of the United States, MIT adopted a European polytechnic university model and stressed laboratory instruction in applied science and engineering. Researchers worked on computers, radar and inertial guidance during World War II and the Cold War. Post-war defense research contributed to the rapid expansion of the faculty and campus under James Killian. The current 168-acre (68.0 ha) campus opened in 1916 and extends over 1 mile (1.6 km) along the northern bank of the Charles River basin.

MIT is a member of the Association of American Universities (AAU). The Institute is traditionally known for its research and education in the physical sciences and engineering, but more recently in biology, economics, linguistics and management as well. MIT is often cited among the world's best universities by various organizations. The MIT Engineers compete in 31 sports, most teams of which

compete in the NCAA Division III's New England Women's and Men's Athletic Conference, whereas the Division I rowing programs compete as part of the EARC and EAWRC.

As of 2017, 88 Nobel laureates, 52 National Medal of Science recipients, 65 Marshall Scholars, 45 Rhodes Scholars, 38 MacArthur Fellows, 34 astronauts, 21 Turing award winners, 16 Chief Scientists of the U.S. Air Force and 6 *Fields Medalists* have been affiliated with MIT. The school has a strong entrepreneurial culture and the aggregated revenues of companies founded by MIT alumni would rank as the eleventh-largest economy in the world

## 2. Harvard University USA



Harvard University is devoted to excellence in teaching, learning, and research, and to developing leaders in many disciplines who make a difference globally. The University, which is based in Cambridge and Boston, Massachusetts, has an enrollment of over 20,000 degree candidates, including undergraduate, graduate, and professional students. Harvard has more than 360,000 alumni around the world.

Harvard faculty are engaged with teaching and research to push the boundaries of human knowledge. For students who are excited to investigate the biggest issues of the 21st century, Harvard offers an unparalleled student experience and a generous financial aid program, with over \$160 million awarded to more than 60% of our undergraduate students. The University has twelve degree-granting Schools in addition to the Radcliffe Institute for Advanced Study, offering a truly global education.

Established in 1636, Harvard is the oldest institution of higher education in the United States.

### **3. Stanford University USA**



Stanford University (officially Leland Stanford Junior University, colloquially the Farm) is a private research university in Stanford, California, in Silicon Valley, 20 miles (30 km) outside of San Jose. Stanford's undergraduate program is the most selective in the United States. Due to its academic strength, wealth, and proximity to Silicon Valley it is often cited as one of the world's most prestigious universities.

The university was founded in 1885 by Leland and Jane Stanford in memory of their only child, Leland Stanford Jr., who had died of typhoid fever at age 15 the previous year. Stanford was a former Governor of California and U.S. Senator; he made his fortune as a railroad tycoon. The school admitted its first students on October 1, 1891, as a coeducational and non-denominational institution.

Stanford University struggled financially after Leland Stanford's death in 1893 and again after much of the campus was damaged by the 1906 San Francisco earthquake. Following World War II, Provost Frederick Terman supported faculty

and graduates' entrepreneurialism to build self-sufficient local industry in what would later be known as Silicon Valley. The university is also one of the top fundraising institutions in the country, becoming the first school to raise more than a billion dollars in a year.

There are three academic schools that have both undergraduate and graduate students and another four professional schools. Students compete in 36 varsity sports, and the university is one of two private institutions in the Division I FBS Pac-12 Conference. It has gained 115 NCAA team championships, the most for a university (one more than UCLA), 483 individual championships, the most in Division I, and has won the NACDA Directors' Cup, recognizing the university with the best overall athletic team achievement, for 22 consecutive years, beginning in 1994–1995. In addition, Stanford students and alumni have won 270 Olympic medals including 139 gold medals.

Stanford faculty and alumni have founded a large number of companies that produce more than \$2.7 trillion in annual revenue, equivalent to the 10th-largest economy in the world. It is the alma mater of 30 living billionaires, 17 astronauts, and 20 Turing Award laureates.[note 1] It is also one of the leading producers of members of the United States Congress. 67 Nobel laureates and 7 *Fields Medalists* have been affiliated with Stanford as students, alumni, faculty or staff.

#### 4. University of Oxford UK



As the oldest university in the English-speaking world, Oxford is a unique and historic institution. There is no clear date of foundation, but teaching existed at Oxford in some form in 1096 and developed rapidly from 1167, when Henry II banned English students from attending the University of Paris.

In 1188, the historian, Gerald of Wales, gave a public reading to the assembled Oxford dons and in around 1190 the arrival of Emo of Friesland, the first known overseas student, set in motion the University's tradition of international scholarly links. By 1201, the University was headed by a magister scholarum Oxonie, on whom the title of Chancellor was conferred in 1214, and in 1231 the masters were recognised as a universitas or corporation.

In the 13th century, rioting between town and gown (townspeople and students) hastened the establishment of primitive halls of residence. These were succeeded by the first of Oxford's colleges, which began as medieval 'halls of residence' or endowed houses under the supervision of a Master. University, Balliol and Merton Colleges, which were established between 1249 and 1264, are the oldest.

Less than a century later, Oxford had achieved eminence above every other seat of learning, and won the praises of popes, kings and sages by virtue of its

antiquity, curriculum, doctrine and privileges. In 1355, Edward III paid tribute to the University for its invaluable contribution to learning; he also commented on the services rendered to the state by distinguished Oxford graduates.

From its early days, Oxford was a centre for lively controversy, with scholars involved in religious and political disputes. John Wyclif, a 14th-century Master of Balliol, campaigned for a Bible in the vernacular, against the wishes of the papacy. In 1530, Henry VIII forced the University to accept his divorce from Catherine of Aragon, and during the Reformation in the 16th century, the Anglican churchmen Cranmer, Latimer and Ridley were tried for heresy and burnt at the stake in Oxford.

The University was Royalist in the Civil War, and Charles I held a counter-Parliament in Convocation House. In the late 17th century, the Oxford philosopher John Locke, suspected of treason, was forced to flee the country.

The 18th century, when Oxford was said to have forsaken port for politics, was also an era of scientific discovery and religious revival. Edmund Halley, Professor of Geometry, predicted the return of the comet that bears his name; John and Charles Wesley's prayer meetings laid the foundations of the Methodist Society.

The University assumed a leading role in the Victorian era, especially in religious controversy. From 1833 onwards The Oxford Movement sought to revitalise the Catholic aspects of the Anglican Church. One of its leaders, John Henry Newman, became a Roman Catholic in 1845 and was later made a Cardinal. In 1860 the new University Museum was the scene of a famous debate between Thomas Huxley, champion of evolution, and Bishop Wilberforce.

From 1878, academic halls were established for women and they were admitted to full membership of the University in 1920. Five all-male colleges first admitted women in 1974 and, since then, all colleges have changed their statutes to admit both women and men. St Hilda's College, which was originally for women only, was the last of Oxford's single sex colleges. It has admitted both men and women since 2008.

During the 20th and early 21st centuries, Oxford added to its humanistic core a major new research capacity in the natural and applied sciences, including medicine. In so doing, it has enhanced and strengthened its traditional role as an international focus for learning and a forum for intellectual debate.

## 5. University of Cambridge UK



The University of Cambridge (informally Cambridge University)[note 1] is a collegiate public research university in Cambridge, England. Founded in 1209 and granted a royal charter by King Henry III in 1231, Cambridge is the second-oldest university in the English-speaking world and the world's fourth-oldest surviving university. The university grew out of an association of scholars who left the University of Oxford after a dispute with the townspeople. The two medieval universities share many common features and are often referred to jointly as "Oxbridge". The history and influence of the University of Cambridge has made it one of the most prestigious universities in the world.

Cambridge is formed from a variety of institutions which include 31 constituent colleges and over 100 academic departments organised into six schools. Cambridge University Press, a department of the university, is the world's oldest publishing house and the second-largest university press in the world. The



university also operates eight cultural and scientific museums, including the Fitzwilliam Museum, as well as a botanic garden. Cambridge's libraries hold a total of around 15 million books, eight million of which are in Cambridge University Library, a legal deposit library.

In the year ended 31 July 2016, the university had a total income of £1.64 billion, of which £462 million was from research grants and contracts. The central university and colleges have a combined endowment of around £6.3 billion, the largest of any university outside the United States. The university is closely linked with the development of the high-tech business cluster known as "Silicon Fen". It is a member of numerous associations and forms part of the "golden triangle" of leading English universities and Cambridge University Health Partners, an academic health science centre.

As of September 2017, Cambridge is ranked the world's second best university by THE World University Rankings, the world's fourth best university by three other ranking tables, and no other institution in the world ranks in the top 10 for as many subjects. The university has educated many notable alumni, including eminent mathematicians, scientists, politicians, lawyers, philosophers, writers, actors and foreign Heads of State. Ninety-eight Nobel laureates, fifteen British prime ministers and *ten Fields medalists* have been affiliated with Cambridge as students, faculty or alumni

## 6. University of California, Berkeley (UCB) USA



The University of California, Berkeley (also referred to as UC Berkeley, Berkeley, and Cal[7]) is a public research university located in Berkeley, California. Founded in 1868, Berkeley is the flagship institution of the ten research universities affiliated with the University of California system and is ranked as one of the world's most prestigious universities and the top public university in the United States.

Established in 1868 as the University of California, resulting from the merger of the private College of California and the public Agricultural, Mining and Mechanical Arts College in Oakland, Berkeley offers approximately 350 undergraduate and graduate degree programs in a wide range of disciplines. The Dwinelle Bill of March 5, 1868 (California Assembly Bill No. 583) said that the "University shall have for its design, to provide instruction and thorough and complete education in all departments of science, literature and art, industrial and profession[al] pursuits, and general education, and also special courses of instruction in preparation for the professions". In the 1960s, Berkeley was particularly noted for the Free Speech Movement as well as the Anti-Vietnam War Movement led by its students.

Berkeley is a founding member of the Association of American Universities and continues to have very high research activity with \$789 million in research and development expenditures in the fiscal year ending June 30, 2015. It also co-manages three United States Department of Energy National Laboratories, including Lawrence Berkeley National Laboratory, Lawrence Livermore National Laboratory and Los Alamos National Laboratory for the U.S. Department of Energy, as well as being home to many world-renowned research institutes and organizations including Mathematical Sciences Research Institute and Space Sciences Laboratory. Through its partner institution University of California, San Francisco (UCSF), Berkeley also offers a joint medical program at the UCSF Medical Center, the top hospital in California, which is also part of the UC system.

Berkeley alumni, faculty and researchers include 94 Nobel laureates (including 34 alumni). They have also won 9 Wolf Prizes, 13 *Fields Medals* (including 3 alumni medalists), 23 Turing Awards (including 11 alumni awardees), 45 MacArthur Fellowships, 20 Academy Awards, 14 Pulitzer Prizes and 207 Olympic medals (117 gold, 51 silver and 39 bronze). Faculty member J. R. Oppenheimer, the "father of the atomic bomb", led the Manhattan project to create the first atomic bomb. Nobel laureate Ernest Lawrence invented the cyclotron, based on which UC Berkeley scientists and researchers, along with Berkeley Lab, have discovered 16 chemical elements of the periodic table – more than any other university in the world. Lawrence Livermore Lab also discovered or co-discovered six chemical elements (113 to 118).

For 2017–18, the Academic Ranking of World Universities (ARWU) ranked Berkeley 5th in the world and 1st among public universities. Berkeley is also ranked 18th internationally among research universities in the Times Higher Education World University Rankings, 6th in the 2017 Times Higher Education World Reputation Rankings. It is additionally ranked 4th internationally (1st among public universities) by U.S. News and World Report'

## 7. Princeton University USA



Princeton University is a private Ivy League research university in Princeton, New Jersey, United States. Founded in 1746 in Elizabeth as the College of New Jersey, Princeton is the fourth-oldest institution of higher education in the United States and one of the nine colonial colleges chartered before the American Revolution. The institution moved to Newark in 1747, then to the current site nine years later, where it was renamed Princeton University in 1896.

Princeton provides undergraduate and graduate instruction in the humanities, social sciences, natural sciences and engineering. It offers professional degrees through the Woodrow Wilson School of Public and International Affairs, the School of Engineering and Applied Science, the School of Architecture and the Bendheim Center for Finance. The university has ties with the Institute for Advanced Study, Princeton Theological Seminary and the Westminster Choir College of Rider University. Princeton has the largest endowment per student in the United States. From 2001 to 2017, Princeton University was ranked either first or second among national universities by U.S. News & World Report, holding the top spot for 15 of those 17 years.

The university has graduated many notable alumni. As of 2017, Princeton has been associated with 43 Nobel laureates, 21 National Medal of Science winners, 14

*Fields Medalists, 5 Abel Prize winners, 10 Turing Award laureates, five National Humanities Medal recipients, 209 Rhodes Scholars, 139 Gates Cambridge Scholars and 126 Marshall Scholars. Two U.S. Presidents, 12 U.S. Supreme Court Justices (three of whom currently serve on the court) and numerous living billionaires and foreign heads of state are all counted among Princeton's alumni body. Princeton has also graduated many prominent members of the U.S. Congress and the U.S. Cabinet, including eight Secretaries of State, three Secretaries of Defense and three of the past five Chairs of the Federal Reserve.*

## 8. ETH Zurich (Swiss Federal Institute of Technology) Switzerland



ETH Zurich (Swiss Federal Institute of Technology in Zurich; German: Eidgenössische Technische Hochschule Zürich) is a science, technology, engineering and mathematics university in the city of Zürich, Switzerland. Like its sister institution EPFL, it is an integral part of the Swiss Federal Institutes of Technology Domain (ETH Domain) that is directly subordinate to Switzerland's Federal Department of Economic Affairs, Education and Research. The school was founded by the Swiss Federal Government in 1854 with the stated mission to educate engineers and scientists, serve as a national center of excellence in science and technology and provide a hub for interaction between the scientific community and industry.

ETH Zurich is consistently ranked amongst the most prestigious universities in Europe for the subjects of engineering and technology. It is ranked 8th in the world in the 2016 edition of the QS World University Rankings, while the QS World

University Rankings by Subject has ranked it as the best university in the world for Earth & Marine Sciences in both 2015/16 and 2016/17's editions.

Twenty-one Nobel Prizes have been awarded to students or professors of the Institute in the past, the most famous of whom was Albert Einstein.

It is a founding member of the IDEA League and the International Alliance of Research Universities (IARU) and a member of the CESAER network.

## 9. University of California, Los Angeles (UCLA) USA



The University of California, Los Angeles (UCLA) is a public research university in the Westwood district of Los Angeles, United States. It became the Southern Branch of the University of California in 1919, making it the second-oldest undergraduate campus of the ten-campus University of California system. It offers 337 undergraduate and graduate degree programs in a wide range of disciplines. UCLA enrolls about 31,000 undergraduate and 13,000 graduate students, and had 119,000 applicants for Fall 2016, including transfer applicants, the most applicants for any American university.

The university is organized into six undergraduate colleges, seven professional schools, and four professional health science schools. The undergraduate colleges are the College of Letters and Science; Henry Samueli School of Engineering and Applied Science (HSSEAS); School of the Arts and Architecture;

Herb Alpert School of Music; School of Theater, Film and Television; and School of Nursing.

Fourteen Nobel laureates, three Fields Medalists, two Chief Scientists of the U.S. Air Force and three Turing Award winners have been faculty, researchers, or alumni. Among the current faculty members, 55 have been elected to the National Academy of Sciences, 28 to the National Academy of Engineering, 39 to the Institute of Medicine, and 124 to the American Academy of Arts and Sciences. The university was elected to the Association of American Universities in 1974.

The Times Higher Education World University Rankings for 2017–2018 ranked UCLA 15th in the world for academics, No.1 US Public University for academics, and 13th in the world for reputation. In 2017, UCLA ranked 12th in the world (10th in North America) by the Academic Ranking of World Universities (ARWU) and 33rd in the 2017–2018 QS World University Rankings. In 2017, the Center for World University Rankings (CWUR) ranked the university 15th in the world based on quality of education, alumni employment, quality of faculty, publications, influence, citations, broad impact, and patents. In 2017–2018, US News & World Report ranked UCLA as the #1 public university in the United States in a tie with its sister campus, UC Berkeley.

UCLA student-athletes compete as the Bruins in the Pac-12 Conference. The Bruins have won 126 national championships, including 114 NCAA team championships, more than any other university except Stanford. UCLA student-athletes, coaches and staff won 251 Olympic medals: 126 gold, 65 silver and 60 bronze. UCLA student-athletes competed in every Olympics since 1920 with one exception (1924), and won a gold medal in every Olympics that the United States participated in since 1932.

## 10. New York University (NYU) USA



New York University (NYU) is a private nonprofit research university based in New York City. Founded in 1831, NYU's main campus is centered in Manhattan, located with its core in Greenwich Village, and campuses based throughout New York City.

NYU is also a worldwide university, operating NYU Abu Dhabi and NYU Shanghai, and centers in Accra, Berlin, Buenos Aires, Florence, London, Madrid, Paris, Prague, Sydney, Tel Aviv, and Washington, D.C.

Among its faculty and alumni are 37 Nobel Laureates, over 30 Pulitzer Prize winners, over 30 Academy Award winners, and hundreds of members of the National Academies of Sciences. Alumni include heads of state, royalty, eminent mathematicians, inventors, media figures, Olympic medalists, CEOs of Fortune 500 companies, and astronauts.

### Mathematical Magic

Courtesy: The manual of mathematical magic

Mathematics and magic may seem a strange combination, but many of the most powerful magical effects performed today have a mathematical basis. Famous magicians such as Derren Brown and David Blaine use mathematics-based tricks in their

shows, but mathematics is also the secret behind the technologies we use, the products we buy and the jobs we will have. Mathematics is the language we use to describe the world around us - it's the basis of all the sciences.



### Multiplication and Addition Doing Fibonacci's Lightning Calculation

On a piece of paper, write the numbers 1 to 10 in a column. You are now all set to amaze with the speed at which you can add ten numbers.

Ask your friend to choose any two two-digit numbers and write the numbers down in the first two spaces of your column, one under the other. Your friend then makes a third number by adding these first two numbers together and writes it below the first two, in effect starting a chain of numbers. They make a fourth number by adding the second and third, a fifth by adding the third and fourth, and so on, until your column of ten numbers is full.

To show how brilliant you are, you can turn away once your friend has understood the idea, say after the seventh number in the list. Now you can't even see the numbers being written.

Meanwhile, with your back turned, you are actually multiplying that seventh number by 11 to get the final answer.

Let's imagine your friend chose 16 and 21 to start with. The list would look like this:

1. 16
2. 21
3. 37
4. 58
5. 95
6. 153
7. 248
8. 401
9. 649
10. 1050

You now turn round and write the sum of all ten numbers straight away! Lightning quick, you say it is 2728. Let them do it slowly on a calculator to show your brilliant mind skills are 100 per cent correct.

The final answer just involves multiplying the seventh number by 11. Why?

Well this chain of numbers where the next term is made by adding the previous two terms is called a Fibonacci sequence. Fibonacci sequences have special mathematical properties that most folk don't know about...

So let's look at the trick. We start with the two numbers  $A$  and  $B$ . The next number is  $A+B$ , the next number is  $B$  added to  $A+B$  which is  $A+2B$  and so on. Going through the number chain we find:

1.  $A$
2.  $B$
3.  $A + B$
4.  $B + (A + B) = A + 2B$
5.  $(A + B) + (A + 2B) = 2A + 3B$
6.  $(A + 2B) + (2A + 3B) = 3A + 5B$
7.  $(2A + 3B) + (3A + 5B) = 5A + 8B$
8.  $(3A + 5B) + (5A + 8B) = 8A + 13B$
9.  $(5A + 8B) + (8A + 13B) = 13A + 21B$
10.  $(8A + 13B) + (13A + 21B) = 21A + 34B$

Adding up all 10 numbers in the chain gives us a grand total of

$55A+88B$  – check it yourself. But look at the seventh number in your column... this line is  $5A+8B$ . It is exactly the total of the chain but divided by 11!

So working backwards, you can get the final total by multiplying the seventh term by 11. And the maths proves this lightning calculation will work for any two starting values  $A$  and  $B$ .

It is up to you to present this trick in such a way that it looks like you are just very, very clever. Which of course you are, as you now know how to use a Fibonacci sequence for magic.

# National Centre for Mathematics

[www.ncmath.org](http://www.ncmath.org)

(A joint centre of TIFR and IIT Bombay)

[www.atmschools.org](http://www.atmschools.org)

## Advanced Training in Mathematics Schools 2018

(Supported by National Board for Higher Mathematics)

<b>Annual Foundation Schools</b>			
<b>Program</b>	<b>Period</b>	<b>Venue</b>	<b>Organizers</b>
AFS - III	2 <sup>nd</sup> – 28 <sup>th</sup> July	KSOM, Kozhikode	M. Manickam T. E. VenkataBalaji A.K. Vijayarajan
AFS - I	3 <sup>rd</sup> – 29 <sup>th</sup> December	IISER Bhopal	Sanjay Kumar Singh Viji Z. Thomas
<b>Instructional Schools for Teachers</b>			
Complex Analysis and Analytical Number Theory	29 <sup>th</sup> January to 10 <sup>th</sup> February	SRTM University, Nanded	K. Srinivas UshaSangale
Topology	14 <sup>th</sup> – 26 <sup>th</sup> May	ISI, Bangalore	Jishnu Biswas ShreedharInamdar AniruddhaNaolekar VaibhavVaish
<b>Advanced Instructional Schools</b>			
Lie Algebras	10 <sup>th</sup> – 29 <sup>th</sup> December	HCRI Allahabad	PunitaBatra Umamaheswaran Arunachalam
Differential Topology	2 <sup>nd</sup> – 21 <sup>st</sup> July	North Eastern Hill University, Sillong	Himadri Kumar Mukerjee AngomTiken Singh
<b>Workshops</b>			
New Directions in PDE Constrained Optimisation	12 <sup>th</sup> to 16 <sup>th</sup> March	IIT Bombay	NeelaNataraj Ajith Kumar
Combinatorial Commutative Algebra	18 <sup>th</sup> to 23 <sup>rd</sup> June	IIT Bombay	Ananthnarayan Hariharan A.V. Jayanthan
Modern aspects of Function Theory, Operator Theory & Operator Algebras	5 <sup>th</sup> to 10 <sup>th</sup> March	ISI Bangalore	Jaydeb Sarkar B.V. RajaramaBhat
<b>Teacher's Enrichment Workshop</b>			
Multivariate Calculus and Linear Algebra	5 <sup>th</sup> to 10 <sup>th</sup> March	R.D University, Jabalpur	SatyaDeo M. Dube Jitendra Kumar Maitra