2015

Mathmation

Compendium of Mathematics Information

Volume - 3 **January - December 2015**

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Students Endaveour
PG & Research Department of Mathematics
Jamal Mohamed College (Autonomous)
College with Potential for Excellence
Accredited with A Grade by NAAC – CGPA 3.6 OUT 4.0
(Affiliated to Bharathidasan University)
Trichy- 20

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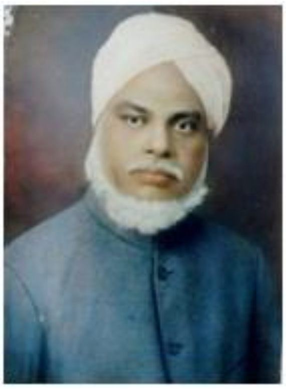
Founders



Hajee M. Jamal Mohamed

Janab N.M.Khajamian Rowther

Our hearty thanks to Janab M.J. Jamal Mohamed Bilal, President, **Dr. A. K. Khaja Nazemudeen,** Secretary and Correspondent, Hajee K.A Khaleel Ahamed, Treasurer, Hajee M.J Jamal Mohamed, Assistant Secretary, Dr. S. Mohamed Salique, Principal, Head and staff members of the Department of Mathematics.

Editorial Board

Introduction

This booklet is the brainchild of motivated Mathematics students & Scholars who wish to disseminate mathematical information regarding the reputed Mathematical Institutions, current events, unsolved problems, Millennium prize problems, puzzles, solutions etc.,

About The College

Jamal Mohamed College was founded in 1951, as an affiliated college to the University of Madras and then affiliated to Bharathidasan University, Tiruchirappalli, since its inception in 1982. The College is administrated by the Society of Jamal Mohamed College. It is established in a sprawling land area of 87 acres as a religious minority institution with the primary objective of providing higher education to the downtrodden and socially backward sections of the society in general and Muslim minority in particular. Over the period of 6 decades, the college was able to scale greater and greater heights and rise to the present status as a multi-faculty institution with 11 UG programmes, 11 PG programmes and 2 M.Phil programmes under Government Aided stream and 8 UG programmes, 15 PG programmes, 15 M.Phil and 15 Ph.D programmes under self-financing stream for men by promoting quality and excellence in higher education due to the sustained efforts and dedicated leadership given by the College Management Committee. The college also offers 12 UG, 14 PG programmes, 15 M.Phil and 15 Ph.D programmes exclusively for women as part of women empowerment during the second shift of the college.

About The Department

The Department of Mathematics of Jamal Mohamed College, was started as one of the earliest departments in the year 1951. B.Sc., and M.Sc Mathematics programmes were started in 1957 and 1963 respectively. The Department was elevated into a research department in the year 2002 by offering M.Phil (Full-Time & Part-Time) and Ph.D (Full-Time & Part-Time) programmes. B.Sc Mathematics (Self-Financing Section) was started for women in 2003. M.Sc Mathematics (Self-Financing Section) was started for women and men in 2005 and 2010 respectively.

In the year 2008, the Department was nationally identified as a Department with potential for award of FIST-Grant by the Ministry of Science and Technology of Government of India.

The Department has highly qualified faculty members actively engaged in teaching, research, continuing education programmes and consultancy. In the last 11 years, the members of the Department have written 10 books, 42 research scholars were awarded Ph.D degrees, published more than 482 research papers in National and International journals and presented many research papers in National and International Conferences.

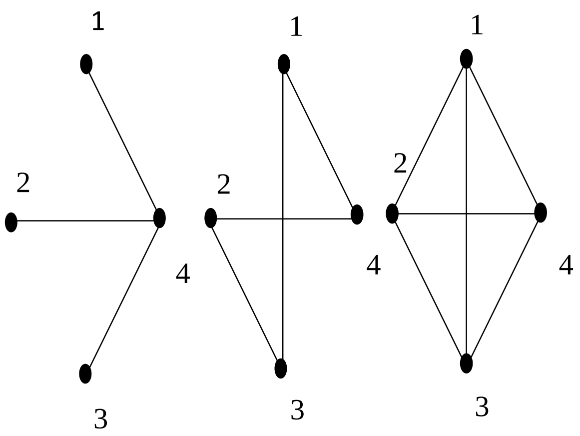
Linear Algebra Applied to Graph Theory

Paul M.Nguyen and Liem Doan

Graph theory has been used for centuries to understand and solve numerous real world problems, including everything from traffic flow to predictions of sports tournaments. Our focus is using graph theory to determine the best methods of traffic flow for any scenario, whether it is networking, electronic circuits, or delivery routes. To approach this problem, we consider the various matrix operations that are well defined on the adjacency matrix, an established method of representing a graph mathematically.

Graph theory can be defined as the mathematical study of the properties of the formal mathematical structures called graphs. A graph is a collection of nodes and edges; a node represents a single entity, an edge provides a connection between exactly two nodes. Edges can be directed, specifying a possibly non-symmetric relationship between two nodes. Nodes in a graph may have no edges (disconnected) or multiple edges.

The adjacency matrix of a simple graph is formed by first numbering the vertices in the graph and then filling the matrix with the element (v_i, v_j) set to 1 there is an edge between them ($v_i$and v_j are adjacent) and 0 otherwise. This convention requires that a simple graph with no loops yields an adjacency matrix with 0 diagonal.



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

An adjacency matrix shows the layout of nodes in a graph.

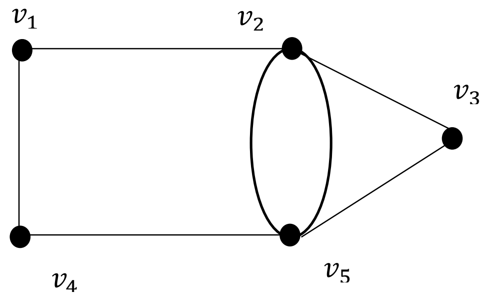
Methods

Our primary method of investigation was to apply various matrix operations to the adjacency matrices of sample graphs in order to determine whether any significant graph-wise transformation resulted.

In order to properly consider the various cases, cliques, connected graphs, disconnected graphs, and special cases in directed graphs were considered.

Directed Graphs

Example: Given a directed graph G as follows

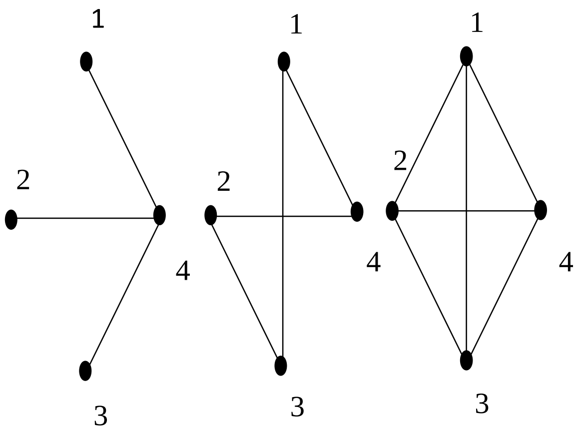


	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	1	0
v_2	0	0	1	0	1
v_3	0	0	0	0	1
v_4	0	0	0	0	1
v_5	0	1	0	0	0

from the chart above, the adjacency matrix for the directed graph is

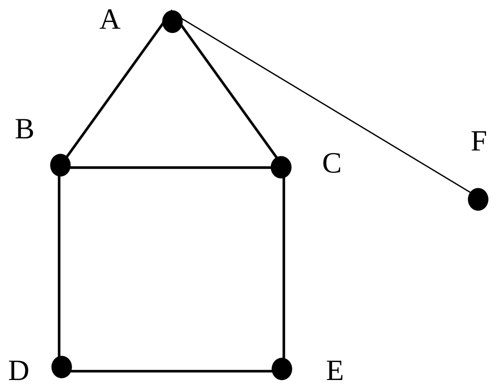
$$\begin{matrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{matrix}$$

Incidence



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Incidence matrices show the relationships between nodes and edges in a graph



$$A(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$S^3 = \begin{pmatrix} 2 & 8 & 2 & 1 & 6 & 7 \\ 8 & 4 & 8 & 4 & 2 & 6 \\ 2 & 8 & 2 & 1 & 6 & 7 \\ 1 & 4 & 1 & 0 & 2 & 2 \\ 6 & 2 & 6 & 2 & 0 & 2 \\ 7 & 6 & 7 & 2 & 2 & 4 \end{pmatrix}$$

Determinant

The determinant of the adjacency matrix of a clique is the degree of every node.

Summary

In studying graph theory we find that we can use adjacency matrices and their components to solve real world applications involving flow rates. These applications range from path efficiency in delivery routes, network redundancy, and ranking predictions.

Using the power properties of an adjacency matrix we are able to see how many different paths there are from one point to another. As shown in our results the powers property of an adjacency matrix we see that the resultant matrix shows the number of paths between any two points. This property of an adjacency matrix is important is important in finding out the best delivery path for a delivery company for instance, a company wants to know the number of paths from point A to point B. in using the adjacency matrix power property they determine that there is 1-step path, and there 2-step paths and from that information the company can choose the most efficient paths.

In addition studying the adjacency matrix of a dominance directed graph we can predict the outcomes by examining the powers of the vertex in the graph. By adding the powers of the adjacency matrix we can identity the ranking of the outcomes of which is to happen first to last. As shown by our research and studies graph theory is very useful in real world applications. So if some ask when will I ever use that in the real world, now you have the answer.

Conclusions

In studying graph theory we find that we can use adjacency matrices and their components to solve real world applications involving flow rates. These applications range from path efficiency in delivery routes, network redundancy, and ranking predictions.

Using the power properties of an adjacency matrix we are able to see how many different paths there are from one point to

another. As shown in our results the powers property of an adjacency matrix we see that the resultant matrix shows that number of paths between any two points. This property of an adjacency matrix is important in finding out the best delivery path for a delivery company. For instance, a company wants to know the number of paths from point A to point B. in using the adjacency matrix power property they determine that there is one 1-step path, and there 2-step paths and from that information the company can choose the most efficient paths.

In addition studying the adjacency matrix of a dominance directed graph we can predict the outcomes by examining the powers of the vertex in the graph. By adding the powers of the adjacency matrix we can identify the ranking of the outcomes of which is to happen first to last. As shown by our research and studies graph theory is very useful in real world applications. So if some ask when will I ever use that in the real world, now you have the answer.

Integration

One of the great achievements of classical geometry was to obtain formulas for the areas and volumes of triangles, spheres, and cones. In this article we study a method to calculate the areas and volumes of these and other more general shapes. The method we develop, called *integration*, is a tool for calculating much more than areas and volumes. The *integral* has many applications in statistics, economics, the sciences, and engineering. It allows us to calculate quantities ranging from probabilities and averages to energy consumption and the forces against a dam's floodgates.

The idea behind integration is that we can effectively compute many quantities by breaking them into small pieces, and then summing the contributions from each small part. We develop the theory of the integral in the setting of area, where it most clearly reveals its nature. We begin with examples involving finite sums. These lead naturally to the question of what happens when more and more terms are summed. Passing to the limit, as the number of terms goes to infinity, then gives an integral. While integration and differentiation are closely connected, we will not see the roles of the derivative and anti derivative emerge.

The nature of their connection, contained in the Fundamental Theorem of Calculus, is one of the most important ideas in calculus.

Area

The area of a region with a curved boundary can be approximated by summing the areas of a collection of rectangles. Using more rectangles can increase the accuracy of the approximation.

Example 1 (Approximating Area)

What is the area of the shaded region *R* that lies above the *x*-axis, below the graph of and between the vertical lines and ? (See Figure 1.) An architect might want to know this area to calculate the weight of a custom window with a shape described by *R*. Unfortunately, there is no simple geometric formula for calculating the areas of shapes having curved boundaries like the region *R*.

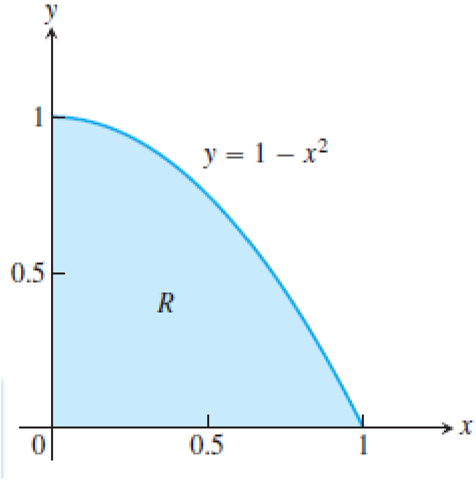


Figure 1 The area of the region *R* cannot be found by a simple geometry formula (Example 1).

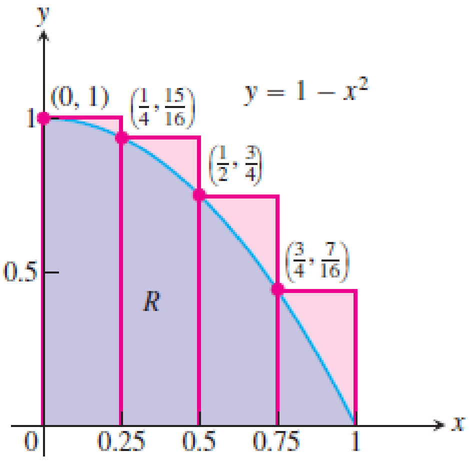


Figure 2 we get an upper estimate of the area of *R* by using Four rectangles give a better upper estimate

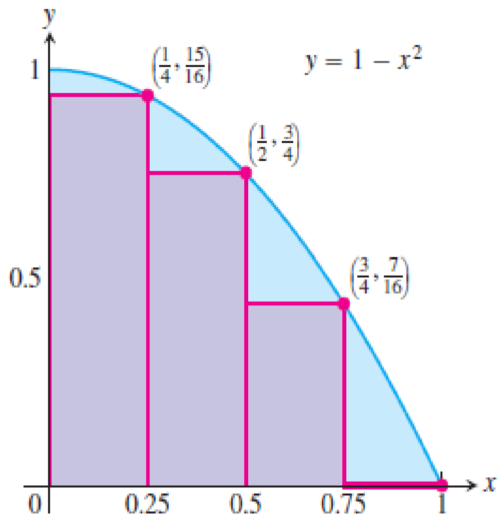
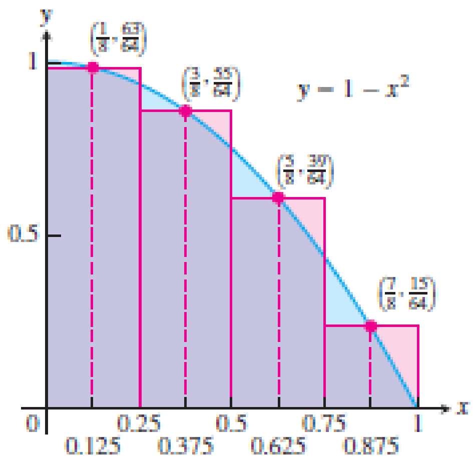


Figure 3 Rectangles contained in *R* give an estimate for the area that undershoots

By considering both lower and upper sum approximations we get not only estimates for the area, but also a bound on the size of the possible error in these estimates since the true value of the area lies somewhere between them. Here the error cannot be greater than the difference $0.78125 - 0.53125 = 0.25$



Yet another estimate can be obtained by using rectangles whose heights are the values
of *f* at the midpoints of their bases (Figure). This method of estimation is called the **midpoint rule** for approximating the area. The midpoint rule gives an estimate that is between a lower sum and an upper sum, but it is not clear whether it overestimates or underestimates the true area. With four rectangles of width $1/4$ as before, the midpoint rule estimates the area of R to be

$$A \approx \frac{63}{64}.\frac{1}{4} + \frac{55}{64}.\frac{1}{4} + \frac{39}{64}.\frac{1}{4} + \frac{15}{64}.\frac{1}{4} = \frac{172}{64}.\frac{1}{4} = 0.671875$$

In each of our computed sums, the interval $[a, b]$ over which the function f is defined was subdivided into *n* subintervals of equal width (also called length) $\Delta x = (b - a)/n$, and f was evaluated at a point in each subinterval: c_1 in the first subinterval c_2 in the second subinterval and so on. The finite sums then all take the form

$$f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + \cdots + f(c_n)\Delta x$$

By taking more and more rectangles, with each rectangle thinner than before, it appears that these finite sums give better and better approximations to the true area of the region *R*.

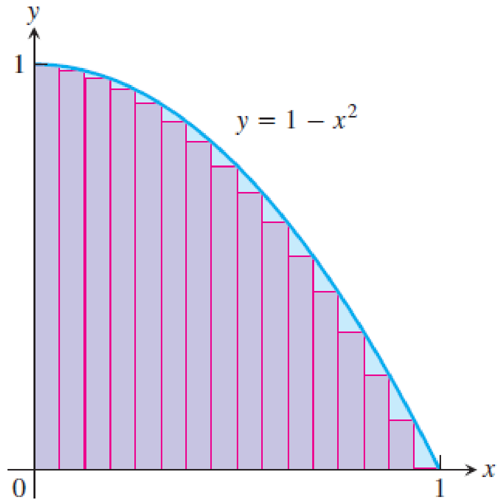


Figure a shows a lower sum approximation for the area of *R* using 16 rectangles of equal width. The sum of their areas is 0.634765625, which appears close to the true area, but is still smaller since the rectangles lie inside *R*.

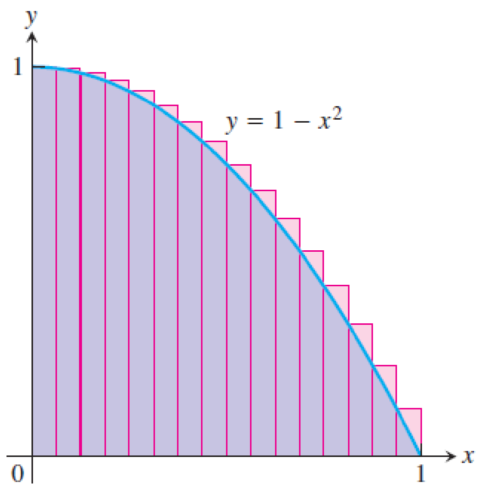


Figure b shows an upper sum approximation using 16 rectangles of equal width. The sum of their areas is 0.697265625, which is somewhat larger than the true area because the rectangles taken together contain *R*. The midpoint rule for 16 rectangles gives a total area approximation of 0.6669921875, but it is not immediately clear whether this estimate is larger or smaller than the true area.

Area Between Intersecting Curves

Example 1 Area Between Intersecting Curves

Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$

Solution

First we sketch the two curves (Figure). The limits of integration are found by solving $y = 2 - x^2$ and $y = -x$ simultaneously for x

$$2 - x^2 = -x$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, \quad x = 2$$

The region runs from $x = -1$ to $x = 2$. The limits of integration are $a = -1$, $b = 2$.

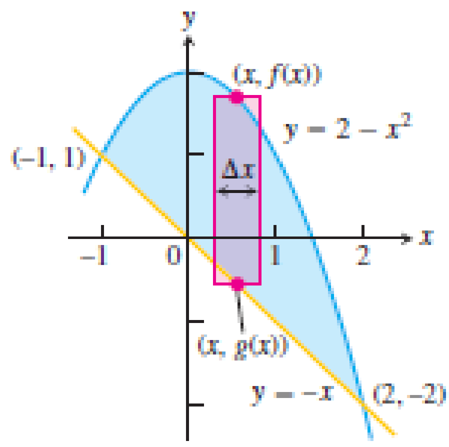


Figure: 1 The region in Example 1 with a typical approximating rectangle.

$$= \left(4 + \frac{4}{2} - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) = \frac{9}{2}$$

The area between the curves is

$$A = \int_a^b [f(x) - g(x)]\, dx = \int_{-1}^{2} [(2 - x^2) - (-x)]\ dx$$

$$= \int_{-1}^{2} (2 + x - x^2)\, dx$$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$$

This is all Greek to me

Small Greek letters used in mathematics

α	alpha	β	beta	γ	gamma	δ	delta
ϵ, ε	epsilon	ζ	zeta	η	eta	θ, ϑ	theta
ι	iota	κ	kappa	λ	lambda	μ	mu
ν	nu	ξ	xi	o	omicron	π, ϖ	pi
ρ, ϱ	rho	σ	sigma	τ	tau	υ	upsilon
ϕ, φ	phi	χ	chi	ψ	psi	ω	omega

Capital Greek letters used in mathematics

B	Beta	Γ	Gamma	Δ	Delta	Θ	Theta
Λ	Lambda	Ξ	Xi	Π	Pi	Σ	Sigma
Υ	Upsilon	Φ	Phi	Ψ	Psi	Ω	Omega

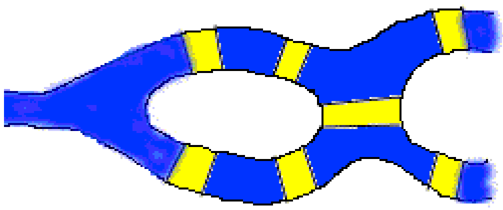
The Beginnings of Topology

Topology is one of the newest branches of mathematics. A simple way to describe topology is as a 'rubber sheet geometry' - topologists study those properties of shapes that remain the same when the shapes are stretched or compressed. The 'Euler number' of a 'network' like the ones presented later in this discussion is an example of a property that does not change when the network is stretched or compressed.

The foundations of topology are often not part of high school math curricula, and thus for many it sounds strange and intimidating. However, there are some readily graspable ideas at the base of topology that are interesting, fun, and highly applicable to all sorts of situations. One of these areas is the topology of networks, first developed by Leonhard Euler in 1735. His work in this field was inspired by the following problem:

The Seven Bridges of Konigsberg

In Konigsberg (modern-day Kaliningrad, Russia), a river ran through the city such that in its center was an island, and after passing the island, the river broke into two parts. Seven bridges were built so that the people of the city could get from one part to another. A crude map of the center of Konigsberg might look like this:



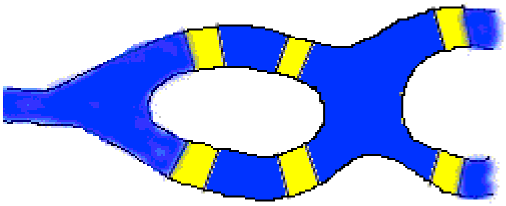
The people wondered whether or not one could walk around the city in a way that would involve crossing each bridge exactly once.

Problem 1

Try it. Sketch the above map of the city on a sheet of paper and try to 'plan your journey' with a pencil in such a way that you trace over each bridge once and only once and you complete the 'plan' with one continuous pencil stroke.

Problem 2

Suppose they had decided to build one fewer bridge in Konigsberg, so that the map looked like this:



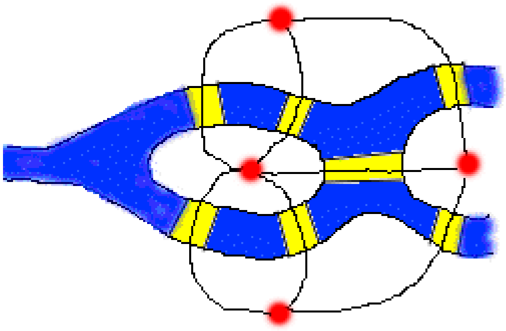
Problem 3

Does it matter which bridge you take away? What if you add bridges? Come up with some maps on your own, and try to 'plan your journey' for each one.

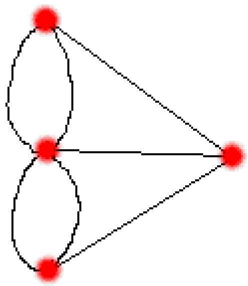
Euler's Solution: The Degree of a Vertex

Euler approached this problem by collapsing areas of land separated by the river into points, which he labelled with capital letters. Modern graph theorists call these *vertices*, and have gone on to represent them and bridges graphically.

For Konigsberg, let us represent land with red dots and bridges with black curves, or *arcs*:



Thus, in its stripped down version, the seven bridges problem looks like this



The problem now becomes one of drawing this picture without retracing any line and without picking your pencil up off the paper. Consider this: all four of the vertices in the above picture have an odd number of arcs connected to them. Take one of these vertices, say one of the ones with three arcs connected to it. Say you're going along, trying to trace the above figure out without picking up your pencil. The first time you get to this vertex, you can leave by another arc. But the next time you arrive, you can't. So you'd better be through drawing the figure when you get there! Alternatively, you could start at that vertex, and then arrive and leave later. But then you can't come back. Thus every vertex with an odd number of arcs attached to it has to be either the beginning or the end of your pencil-path. So you can only have up to two 'odd' vertices! Thus it is impossible to draw the above picture in one pencil stroke without retracing.

The Generalization to Graph Theory

Euler went on to generalize this mode of thinking, laying a foundation for graph theory. Using modern vocabulary, we make the following definitions and prove a theorem:

Definition: A network is a figure made up of points (vertices) connected by non-intersecting curves (arcs).

Definition: A vertex is called odd if it has an odd number of arcs leading to it, other wise it is called even.

Definition: An Euler path is a continuous path that passes through every arc once and only once.

Theorem: If a network has more than two odd vertices, it does not have an Euler path.

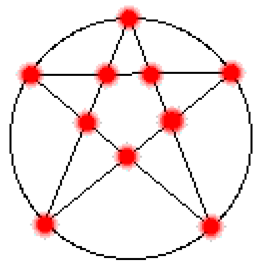
Euler also proved this:

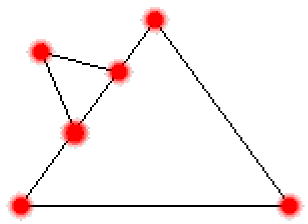
Theorem: If a network has two or zero odd vertices, it has at least one Euler path. In particular, if a network has exactly two odd vertices, then its Euler paths can only start on one of the odd vertices, and end on the other.

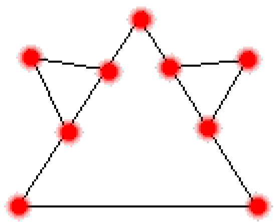
Problems

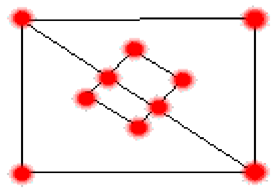
Problem 4:

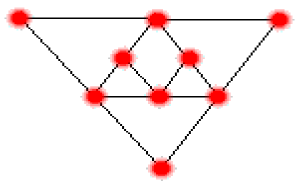
For each of the networks below, determine whether it has an Euler path. If it does, find one.

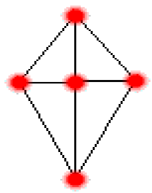






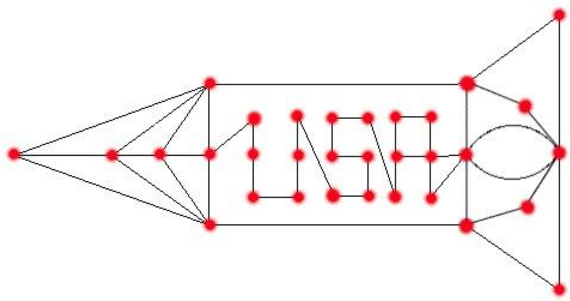






Problem 5:

Without finding (or trying to find) an Euler path, determine whether the network below has any Euler paths.



Maths - where is it used in Engineering

Complex numbers (include De Moivre's theorem)

Engineering Applications

Electrical Engineering (A.C.Circuits)

Resistors, inductors, capacitors, power engineering, analysis of electric & magnetic fields and their interactions with materials and structures.

Electronics

Digital signal processing, image processing

Mechanical / Civil Engineering

Fluid flow, stress analysis.

Sports and Exercise Engineering / Biomedical Engineering:

Signal processing and analysis, power meters, heart rate monitors.

Energy systems engineering:

Design of control systems to protect ocean energy converters at sea.

Matrices and determinants (incl. eigenvalues and eigenvalues)

Civil Engineering:

Traffic engineering and modelling, structural engineering (trusses), structural engineering.

Electrical Engineering (A.C. Circuits)

Electrical networks

Electronic Engineering & IT:

Computer graphics (zoom, rotations, transformations, animation etc), Google search algorithms, Image analysis including facial recognition; simplification of complex data sets, advanced data and systems modelling, digital communications

Mechanical Engineering:

Mechanics, representation of stress and strain

Energy Systems Engineering:

Predicting emissions of pollutants from next generation jet engines.

Laplace transforms (solving differential equations)

Biomedical Engineering:

Analysis of defibrillator systems, drug delivery, fluid flow

Electrical Engineering:

Circuits, power systems (generators), feedback loops (e.g. control of aircraft systems)

Civil Engineering:

Structural design (earthquake engineering)

Mechanical Engineering:

Mechanics of vibrations, fluid flow

Energy Systems Engineering:

Control of battery charging and discharging in electric cars

Statistics and Probability

Civil Engineering:

Flood modelling, water / wastewater treatment

Electronic Engineering & IT:

Designing and modelling the internet

Biomedical Engineering:

Measuring performance of drugs, catheters, prostheses etc

Energy systems Engineering / Civil Engineering:

Wind energy generation, wave height prediction

Electronic Engineering & IT:

Failure rates for semiconductor devices; behaviour of semiconductor materials and structures; image analysis; data compression; digital communications techniques and error correction

Prime Numbers

One of the most important and beautiful fields of mathematics is number theory - the study of numbers and their properties. Despite the fact that mathematicians have been studying numbers for as long as humans have been able to count, the field of number theory is far from being outdated; some of the most exciting and important problems in mathematics today have to do with the study of numbers. In particular, prime numbers are of great interest.

Definition: A number p is prime if it is a positive integer greater than 1 and is divisible by no other positive integers other than 1 and itself.

Positive integers greater than 1 that aren't prime are called composite integers.

Examples: 2,3, and 5 are prime. 6 is composite. All positive integers n have at least one prime divisor: if n is prime, then it is its own prime divisor. If n is composite, and one factors it completely, one will have reduced n to prime factors.

Examples: 6=3*2, 18=3*3*2, 48=6*8=2*3*2*2*2

The following theorem was proved eloquently by Euclid.

Theorem: There are infinitely many prime numbers.

Proof:

Suppose the opposite, that is, that there are a finite number of prime numbers. Call them $p_1, p_2, p_3, p_4,....,p_n$. Now consider the number

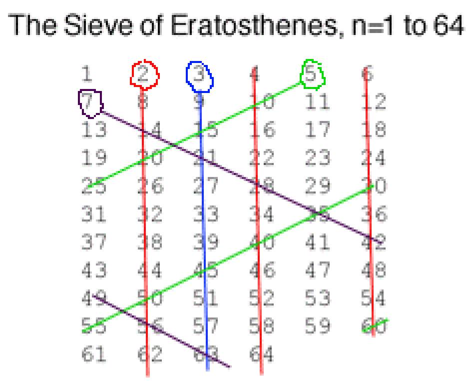
$(p_1*p_2*p_3*...*p_n)+1$

Every prime number, when divided into this number, leaves a remainder of one. So this number has no prime factors (remember, by assumption, it's not prime itself). This is a contradiction. Thus there must, in fact, be infinitely many primes.

So, that proves that we'll never find all of the prime numbers because there's an infinite number of them. But that hasn't stopped mathematicians from looking for them, and for asking all kinds of neat questions about prime numbers.

How do you find Prime Numbers?

Good question. It's one mathematicians are still trying to answer. The simplest method was developed by Eratosthenes in the 3rd century B.C. Here's how it works: Suppose we want to find all the prime numbers between 1 and 64. We write out a table of these numbers, and proceed as follows. 2 is the first integer greater than 1, so it is obviously prime. We now cross out all multiples of two. The next number that we haven't crossed out is 3. We circle it and cross out all its multiples. The next non-crossed-out number is 5, so we circle it and cross out all its multiples. We only have to do this for all numbers less than the square root of our upper limit (in this case sqrt(64)=8) since any composite number in the table must have at least one factor less than the square root of the upper limit. What's left after this process of elimination is all the prime numbers between 1 and 64.



Unfortunately, this method is rather time-consuming when the numbers you are looking for are much larger.

More Prime Number Theory

Mathematicians have developed a great amount of theory concerning prime numbers. Here's a taste of it:

Question: how far are prime numbers from each other? Sometimes, only 2 integers apart, like 41 and 43. Although there is a lot of evidence to suggest it, no one has proved that there are an infinite number of "twin primes."

In general, however, primes get more spread out as they get larger. By how much? In 1896 Charles de la Vallee-Poussin and Jacques Hadamard proved the **Prime Number Theorem**, which states:

Let Pr(x) be the number of prime numbers less than x. Then the ratio of Pr(x) to (x/ln(x)) approaches 1 as x grows without bound.

What this implies is that if n is a prime number, the distance to the next prime number is, on average, approximately ln(n).

The Goldbach Conjecture

In a letter to Leonard Euler in 1742, Christian Goldbachconjectured that every positive even integer greater than 2 can be written as the sum of two primes. Though computers have verified this up to a million, no proof has been given. Since Goldbach's time, however, his idea has been broken down into the 'strong' Goldbach conjecture - his original claim - and the 'weak' Goldbach conjecture, which claims that every odd number greater than 7 can be expressed as the sum of three odd primes.

What Is the Difference between a Theorem, a Lemma, and a Corollary

Prof. Dave richeson

(1) **Definition**—a precise and unambiguous description of the meaning of a mathematical term. It characterizes the meaning of a word by giving all the properties and only those properties that must be true.

(2) **Theorem**—a mathematical statement that is proved using rigorous mathematical reasoning. In a mathematical paper, the term theorem is often reserved for the most important results.

(3) **Lemma**—a minor result whose sole purpose is to help in proving a theorem. It is a stepping stone on the path to proving a theorem. Very occasionally lemmas can take on a life of their own (Zorn's lemma, Urysohn's lemma, Burnside's lemma, Sperner's lemma).

(4) **Corollary**—a result in which the (usually short) proof relies heavily on a given theorem (we often say that "this is a corollary of Theorem A").

(5) **Proposition**—a proved and often interesting result, but generally less important than a theorem.

(6) **Conjecture**—a statement that is unproved, but is believed to be true (Collatz conjecture, Goldbach conjecture, twin prime conjecture).

(7) **Claim**—an assertion that is then proved. It is often used like an informal lemma.

(8) **Axiom/Postulate**—a statement that is assumed to be true without proof. These are the basic building blocks from which all theorems are proved (Euclid's five postulates, Zermelo-Frankel axioms, Peano axioms).

(9) **Identity**—a mathematical expression giving the equality of two (often variable) quantities (trigonometric identities, Euler's identity).

(10) **Paradox**—a statement that can be shown, using a given set of axioms and definitions, to be both true and false. Paradoxes are often used to show the inconsistencies in a flawed theory (Russell's paradox). The term paradox is often used informally to describe a surprising or counterintuitive result that follows from a given set of rules (Banach-Tarski paradox, Alabama paradox, Gabriel's horn).

Algebra in Everyday Life

We use algebra quite frequently in our everyday lives, and without even realizing it! We not only use algebra, we actually need algebra, to solve most of our problems that involves calculations.

Examples of using algebra in everyday life

Here are some simple examples that demonstrate the relevance of algebra in the real world.

Example 1: Going shopping

You purchased 10 items from a shopping plaza. And now you need plastic bags to carry them home. If each can hold only 3 items, how many plastic bags you will need to accommodate 10 items.

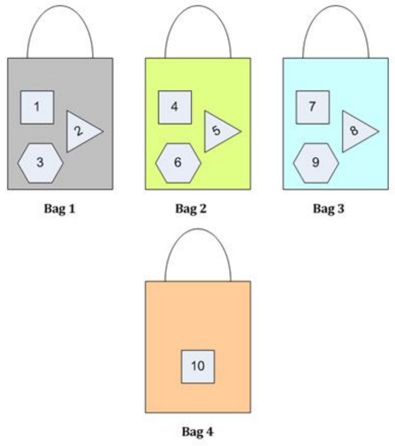
Answer:

$$\frac{10\ items}{3\ items\ /\ bag} = 3.33\ bags\ \approx 4\ bags$$

Explanation:

The figure below illustrates the problem:

The different shapes inside the bags denote different items purchased. The number depicts the item number.



We use simple algebraic formula $\frac{x}{y}$ to calculate the number of bags.

x = Number of items purchased = 10

y = Capacity of 1 bag = 3

Hence,

$$\frac{10}{3} = 3.33\ bags\ \approx 4\ bags$$

So, we need 4 shopping bags to put 10 items.

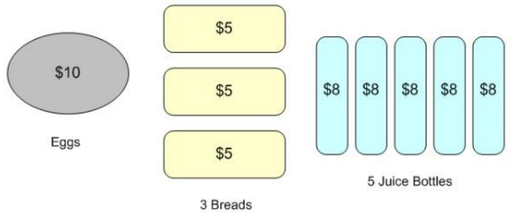
Example 2: Calculating Grocery expense

You have to buy two dozen eggs at $10, three breads (each bread is $5), and five bottles of juice (each bottle is $8). How much money you will need to take to the grocery store?

Answer: $ 65

Explanation: The figure below shows the three items in different shapes and colors.

This will help your mind to calculate faster.



We will use algebra to solve the problem easily and quickly.

The prices are

a = Price of two dozen eggs = $10

b = Price of one bread = $5

c = Price of one bottle of juice= $8

Money needed = a + 3b + 5c

Money needed = $10 + 3($5) + 5($8) = $10 + $15 + $40

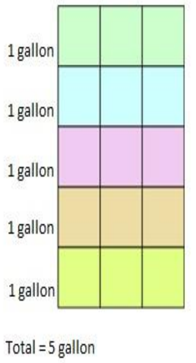
= $65

Example 3: Filling up the gas tank

You need to fill the gas tank but you have only $ 15 in your pocket. If the price of the gas is $ 3 a gallon, how many gallons can you buy?

Answer: 5 gallons

Explanation: In the below diagram, each block represents $1, and each row is a bundle of $3, which is used to buy 1 gallon of gas.



We use simple algebraic formula, $\frac{x}{y}$ to calculate the total gallons that can be bought.

x = Money in your pocket= $15

y = Price of 1 gallon of gas= $3

Hence,

$\frac{\$15}{\$3}$= 5 gallon

So, with $15 we can buy 5 gallons of gas.

Applications of Calculus

With calculus, we have the ability to find the effects of changing conditions on a system. By studying these, you can learn how to control a system to make it do what you want it to do. Because of the ability to model and control systems, calculus gives us extraordinary power over the material world.

Calculus is the language of engineers, scientists, and economists. The work of these professionals has a huge impact on our daily life - from your microwaves, cell phones, TV, and car to medicine, economy, and national defense.

Credit card companies use calculus to set the minimum payments due on credit card statements at the exact time the statement is processed by considering multiple variables such as changing interest rates and a fluctuating available balance.

Biologists use differential calculus to determine the exact rate of growth in a bacterial culture when different variables such as temperature and food source are changed. This research can help increase the rate of growth of necessary bacteria, or decrease the rate of growth for harmful and potentially threatening bacteria.

An **electrical engineer** uses integration to determine the exact length of power cable needed to connect two substations that are miles apart. Because the cable is hung from poles, it is constantly curving. Calculus allows a precise figure to be determined.

An **architect** will use integration to determine the amount of materials necessary to construct a curved dome over a new sports arena, as well as calculate the weight of that dome and determine the type of support structure required.

Space flight engineers frequently use calculus when planning lengthy missions. To launch an exploratory probe, they must consider the different orbiting velocities of the Earth and the planet the probe is targeted for, as well as other gravitational influences like the sun and the moon. Calculus allows each of those variables to be accurately taken into account.

Statisticians will use calculus to evaluate survey data to help develop business plans for different companies. Because a survey involves many different questions with a range of possible answers, calculus allows a more accurate prediction for appropriate action.

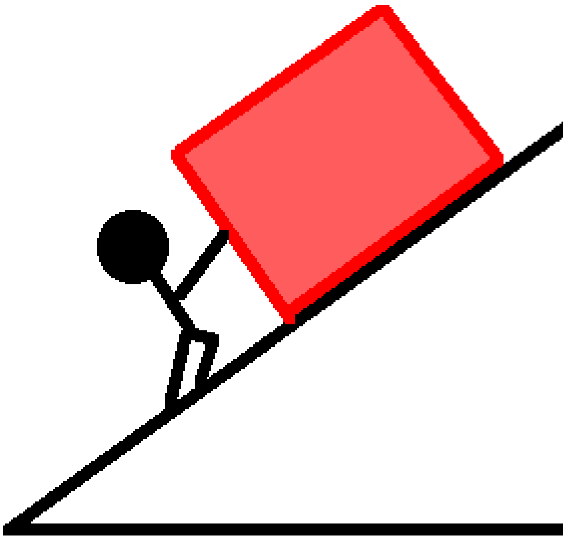
A **physicist** uses calculus to find the center of mass of a sports utility vehicle to design appropriate safety features that must adhere to federal specifications on different road surfaces and at different speeds.

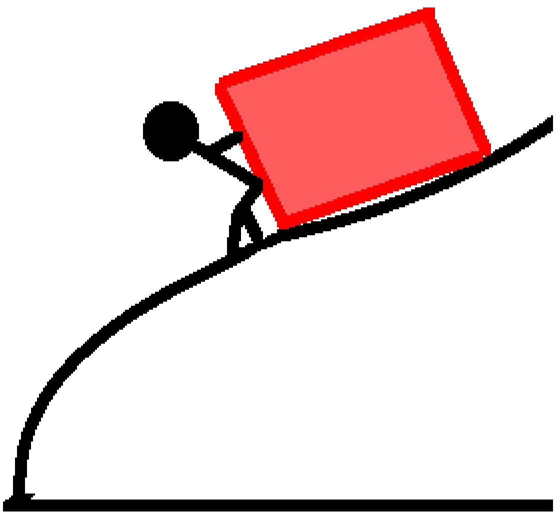
An **operations research analyst** will use calculus when observing different processes at a manufacturing corporation. By considering the value of different variables, they can help a company improve operating efficiency, increase production, and raise profits.

A **graphics artist** uses calculus to determine how different three-dimensional models will behave when subjected to rapidly changing conditions. This can create a realistic environment for movies or video games.

Obviously, a wide variety of careers regularly use calculus. Universities, the military, government agencies, airlines, entertainment studios, software companies, and construction companies are only a few employers who seek individuals with a solid knowledge of calculus. Even doctors and lawyers use calculus to help build the discipline necessary for solving complex problems, such as diagnosing patients or planning a prosecution case. Despite its mystique as a more complex branch of mathematics, calculus touches our lives each day, in ways too numerous to calculate.

Difference between Calculus and Other Math Subjects



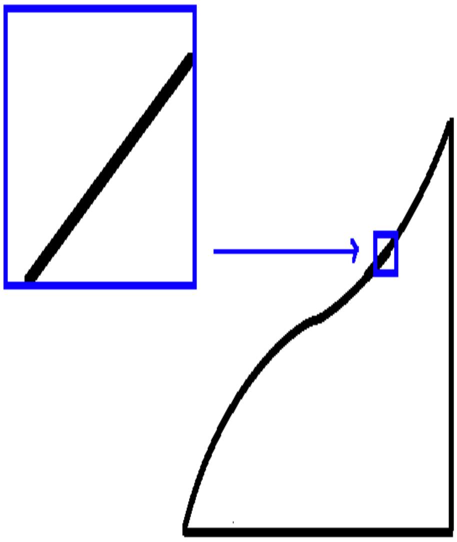


On the left, a man is pushing a crate up a straight incline. On the right, a man is pushing the same crate up a curving incline. The problem in both cases is to determine the amount of energy required to push the crate to the top. For the problem on the left, you can use algebra and trigonometry to solve the problem. For the problem on the right, you need calculus. Why do you need calculus with the problem on the right and not the left?

This is because with the straight incline, the man pushes with an unchanging force and the crate goes up the incline at an unchanging speed. With the curved incline on the right, things are constantly changing. Since the steepness of the incline is constantly changing, the amount of energy expended is also changing. This is why calculus is described as "the mathematics of change". Calculus takes regular rules of math and applies them to evolving problems.

With the curving incline problem, the algebra and trigonometry that you use is the same, the difference is that you have to break up the curving incline problem into smaller chunks and do each chunk separately. When zooming in on a small portion of the curving incline, it looks as if it is a straight line:

Then, because it is straight, you can solve the small chunk just like the straight incline problem. When all of the small chunks are solved, you can just add them up.



This is basically the way calculus works - it takes problems that cannot be done with regular math because things are constantly changing, zooms in on the changing curve until it becomes straight, and then it lets regular math finish off the problem. What makes calculus such a brilliant achievement is that it actually zooms in infinitely. In fact, everything you do in calculus involves infinity in one way or another, because if something is constantly changing, it is changing infinitely from each infinitesimal moment to the next. All of calculus relies on the fundamental principle that you can always use approximations of increasing accuracy to find the exact answer. Just like you can approximate a curve by a series of straight lines, you can also approximate a spherical solid by a series of cubes that fit inside the sphere.

Top 10 Universities for Mathematics in the World

Based on the QS World University Rankings by Subject 2015

Harvard University

Harvard University is devoted to excellence in teaching, learning, and research, and to developing leaders in many disciplines who make a difference globally. Harvard faculty are engaged with teaching and research to push the boundaries of human knowledge. For students who are excited to investigate the biggest issues of the 21st century, Harvard offers an unparalleled student experience and a generous financial aid program, with over $160 million awarded to more than 60% of our undergraduate students. The University has twelve degree-granting Schools in addition to the Radcliffe Institute for Advanced Study, offering a truly global education. Established in 1636, Harvard is the oldest institution of higher education in the United States. The University, which is based in Cambridge and Boston, Massachusetts, has an enrollment of over

20,000 degree candidates, including undergraduate, graduate, and professional students. Harvard has more than 360,000 alumni around the world.

University of Cambridge

The University of Cambridge is rich in history - its famous Colleges and University buildings attract visitors from all over the world. But the University's museums and collections also hold many treasures which give an exciting insight into some of the scholarly activities, both past and present, of the University's academics and students. The University of Cambridge is one of the oldest universities in the world and one of the largest in the United Kingdom. Its reputation for outstanding academic achievement is known world-wide and reflects the intellectual achievement of its students, as well as the world-class original research carried out by the staff of the University and the Colleges. Its reputation is endorsed by the Quality Assurance Agency and by other external reviewers of learning and teaching, such as External Examiners. These high standards are the result of both the learning opportunities offered at Cambridge and by its extensive resources, including libraries, museums and other collections. Teaching consists not only of lectures, seminars and practical classes led by people who are world experts in their field, but also more personalised teaching arranged through the Colleges. Many opportunities exist for students to interact with scholars of all levels, both formally...

University of Oxford

Oxford is the oldest university in the English-speaking world and lays claim to nine centuries of continuous existence. As an internationally renowned centre for teaching and research, Oxford attracts students and scholars from across the globe, with almost a quarter of our students from overseas. More than 130 nationalities are represented among a student population of over 18,000. Oxford is a collegiate university, with 39 self-governing colleges related to the University in a type of federal system. There are also seven Permanent Private Halls, founded by different Christian denominations. Thirty colleges and all halls admit students for both undergraduate and graduate degrees. Seven other colleges are for graduates only; one has Fellows only, and one specializes in part-time and continuing education. There is no clear date of foundation, but teaching existed at Oxford in some form in 1096 and developed rapidly from 1167, when Henry II banned English students from attending the University of Paris. Oxford is one of Europe's most innovative and entrepreneurial universities. Drawing on an 800-year tradition of discovery and invention, modern Oxford leads the way in creating jobs, wealth, skills and innovation for the 21st century. The leading UK university for knowledge transfer and commercial spin-outs.

Massachusetts Institute of Technology

The mission of MIT is to advance knowledge and educate students in science, technology, and other areas of scholarship that will best serve the nation and the world in the 21st century. The Institute is committed to generating, disseminating, and preserving knowledge, and to working with others to bring this knowledge to bear on the world's great challenges. MIT is dedicated to providing its students with an education that combines rigorous academic study and the excitement of discovery with the support and intellectual stimulation of a diverse campus community. We seek to develop in each member of the MIT community the ability and passion to work wisely, creatively, and effectively for the betterment of humankind. The Institute admitted its first students in 1865, four years after the approval of its founding charter. The opening marked the culmination of an extended effort by William Barton Rogers, a distinguished natural scientist, to establish a new kind of independent educational institution relevant to an increasingly industrialized America. Rogers stressed the pragmatic and practicable. He believed that professional competence is best fostered by coupling teaching and research and by focusing attention on real-world problems. Toward this end, he pioneered the development of the teaching.

Stanford University

Stanford University, founded in 1885, is recognized as one of the world's leading research and teaching institutions, with one of the most renowned faculties in the nation. Stanford students—men and women of all races, ethnicities and ages — are distinguished by their love of learning and desire to contribute to the greater community. Stanford University offers its students a remarkable range of academic and extracurricular activities. We are committed to offering an education that is unrivaled among research universities. In this community of scholars, there is no greater objective than being at the edge of a field and advancing the frontier of knowledge. We believe that collaboration across disciplines will be key to future advances and are pursuing multidisciplinary initiatives in the areas of biosciences, the environment and international affairs. As a research and teaching university, we offer both undergraduate and graduate students opportunities to work closely with faculty and researchers. The pioneering spirit that inspired Jane and Leland Stanford to start this university more than a century ago and that helped build Silicon Valley at the doorstep of the campus encourages boldness in everything we do - whether those efforts occur in the library, in the classroom, in a laboratory,.

University of California

The University of California, Berkeley is one of the world's leading academic institutions. Widely known as "Cal," the campus is renowned for the size and quality of its libraries and laboratories, the scope of its research and publications, and the distinction of its faculty and students. National rankings consistently place Berkeley's undergraduate and graduate programs among the very best in a variety of disciplines. Our high-acclaimed faculty currently includes: 7 Nobel Laureates, 225 members of the Academy of Arts & Sciences, 131 members of the National Academy of Science, 87 members of the National Academy of Engineering, a Poet Laureate Emeritus of the United States, and 141 Guggenheim Fellows, more than any other university in the country. It was here that two professors discovered plutonium in 1941 as well as numerous other elements, including berkelium and californium. Berkeley faculty are quoted daily in newspapers and journals throughout the world as experts in their fields. But Berkeley is also about extraordinary students! While most of our 22,800 undergraduates are Californians, every state and more than 100 foreign countries are represented on campus. The student body can best be characterized by its talent and its diversity; in fact, there is no single.

Princeton University

Princeton is the fourth-oldest college in the United States. The ambience of its earliest days is palpable in historic landmarks on campus, most notably Nassau Hall, which in 1783 was the temporary capitol of the United States. From such a distinctive beginning grew something great -- a community of learning that continues to evolve, providing abundant opportunities to talented students from around the world. As a research university, it seeks to achieve the highest levels of distinction in the discovery and transmission of knowledge and understanding, and in the education of graduate students. At the same time, Princeton is distinctive among research universities in its commitment to undergraduate teaching. The University provides its students with academic, extracurricular and other resources -- in a residential community committed to diversity in its student body, faculty and staff -- that help them achieve at the highest scholarly levels and prepare them for positions of leadership and lives of service in many fields of human endeavor. Through the scholarship and teaching of its faculty, and the many contributions to society of its alumni, Princeton seeks to fulfill its informal motto: “Princeton in the Nation’s Service and in the Service of All Nations.

University of California

A Brief History of UCLA University of California, Los Angeles With only 11,000 inhabitants in 1880, the pueblo of Los Angeles convinced the state government to establish a State Normal School in Southern California. Enthusiastic citizens contributed between $2 and $500 to purchase a site, and on August 29, 1882, the Los Angeles Branch of the State Normal School welcomed its first students in a Victorian building that had been erected on the site of an orange grove. By 1914 Los Angeles had grown to a city of 350,000, and the school moved to new quarters - a Hollywood ranch off a dirt road that later became Vermont Avenue. In 1919, the school became the Southern Branch of the University of California and offered two years of instruction in Letters and Science. Third- and fourth-year courses were soon added; the first class of 300 students was graduated in 1925, and by 1927 the Southern Branch had earned its new name: University of California at Los Angeles. (The name was changed again in 1958 to University of California, Los Angeles.) Continued growth mandated the selection of a site that could support a larger campus, and in 1927, ground was broken in.

ETH Zurich - Swiss Federal Institute of Technology

Consistently ranked the top university in continental Europe, ETH Zurich, the Swiss Federal Institute of Technology, is a leading player in research and education in Switzerland and worldwide. ETH Zurich's 16 departments offer Bachelor, Master and Doctoral programmes in engineering and natural sciences. The language of instruction in the Bachelor programmes is German, whereas English is the common language on the graduate level. All degree programmes provide a solid scientific foundation combined with outstanding all-round skills, equipping ETH graduates with the abilities and flexibility needed for a career in industry, business or the public sector, as entrepreneur or scientist. The international outlook - 70% of the professors have been recruited from abroad - and the excellent teaching and research infrastructure make

ETH Zurich the ideal place for creative personalities. The links with business and industry are very close, Zurich being the economic center of Switzerland and home to numerous international companies. And beyond world-class education, Zurich also offers many other quality-of-life highlights. Zurich has a metropolitan flair, excellent sports facilities, an extensive range of cultural and recreational offerings - and a very vibrant nightlife.

University of Chicago

The University of Chicago was founded in 1890 by the American Baptist Education Society and oil magnate John D. Rockefeller, who later described the University of Chicago as “the best investment I ever made.” The land for the new university, in the recently annexed suburb of Hyde Park, was donated by Marshall Field, owner of the Chicago department store that bears his name. William Rainey Harper, the first president, imagined a university that would combine an American-style undergraduate liberal arts college with a German-style graduate research university. The University of Chicago quickly fulfilled Harper's dream, becoming a national leader in higher education and research. Frederick Rudolph, professor of history at Williams College, wrote in his 1962 study, The American College and University: A History, “No episode was more important in shaping the outlook and expectations of American higher education during those years than the founding of the University of Chicago, one of those events in American history that brought into focus the spirit of an age.” One of Harper's curricular innovations was to run classes all year round, and to allow students to graduate at whatever time of year they completed their studies. Appropriately enough, the first class was held.

National Centre for Mathematics

www.ncmath.org

(A joint centre of TIFR and IIT Bombay)

www.atmschools.org

Advanced Training in Mathematics Schools 2016

(Supported by National Board for Higher Mathematics)

Annual Foundation Schools			
Program	**Period**	**Venue**	**Organizers**
AFS - I	1st - 28th Dec	IIT Guwahati	Anupam Saikia Rupam Barman
AFS - I	5th - 31st Dec	IISER Thiruvananthapuram	Viji Thomas A.R. Shastri
AFS - I	5th - 31st Dec	HRI Allahabad	N. Raghavendra Archana Morye
AFS - II	2nd - 28th May	Bhaskaracharya Pratishthane, Pune.	S.A. Katre V.V. Acharya
AFS - II	6th June - 1st July	KSOM Kozhikode	M. Manickam A.J. Parameswaran
AFS - III	20th June - 16th July	IISER Thiruvananthapuram	Viji Thomas A.R. Shastri
Instructional Schools for Lecturers			
Field Theory	2nd - 14th May	University of Kashmir, Srinagar	Amin Sofi Dinesh Khurana
Algebraic Number Theory	Oct	S.P. Pune University, Pune	S.A. Katre S.D. Adhikari
Fourier Analysis	Nov	Bhaskaracharya Pratishthana, Pune	V.M. Sholapurkar
Topology	9th - 21st May	IIT Bombay	Rekha Santhanam Swagata Sarkar
Advanced Instructional Schools			
Operator Theory / Algebra	1st - 21st, Feb	IMsc Chennai	V.S. Sunder Vijay Kodiyalam
Matrix Analysis	2nd - 21st, May	Shiv Nadar University, Greater Noida	Rajendra Bhatia Priyanka Grover
Optimization	9th - 21st, May	IIT Bombay	K. Sureshkumar Ankur Kulkarni Mallikarjun Rao
Differential Geometry	9th - 28th, May	IIT Bombay	M.S. Raghunathan Akhil Ranjan

Algebraic Geometry	16th May - 4th, Jun	ISI, Bangalore	Jishnu Biswas S. Inamdar A. Tripathi
Stochastic Process	6th - 25th, Jun	ISI, Kolkata	Alok Goswami
Algebraic Topology	13thJun - 2nd , Jul	NEHU, Shillong	Himadri Mukherjee
Mathematical Programming	4th - 23rd, Jul	Central University of Tamil Nadu, Thiruvarur	R. B. Bapat A. Chandrashekaran
Class Field Theory	5th - 23rd, Dec	CMI, Chennai	M. Ram Murty P. Rath Sanoli Gun
Harmonic Analysis	5th - 24th, Dec	KSOM Kozhikode	E. K. Narayanan M. Manickam
PDE and Mechanics		TIFR, CAM	Vasudev Murty
Workshops			
Applied Probability	04th - 08th, Jan	IIT Bombay	V.S. Borkar K. Suresh Kumar Rajesh Sundaresan
PDE & Mechanics	01st- 06th, Feb	KSOM, Kozhikode	M. Manickam Veerappa Gowda
Probability & Representation Theory	07th - 12th, Mar	IMSc, Chennai	K.N. Raghavan Arvind Ayyer
Teacher's Enrichment Workshop			
Rings and Fields, Complex Analysis, Geometry of Curves	04th - 09th, Jan	Presidency University, Kolkata	Shubhabrata Das Mahuya Datta
Linear Algebra, Analysis of several variables, Probability Theory	01st - 06th, Feb	Brahmananda Keshab Chandra College (BKC College)	-Neeta Pandey -Mahuya Datta
		Delhi	Abhay Bhatt
		Chennai	K. Srinivas K.N. Raghavan
		Bangalore	Gadadhar Misra
		Allahabad	D.Surya Ramana
		Guwahati	Rafikul Alam
		IISER Mohali	Amit Kulshrestha Dinesh Khurana
		Srinagar	Amin Sofi