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Jamal Mohamed College (Autonomous)
College with Potential for Excellence
Accredited with A Grade by NAAC – CGPA 3.6 OUT 4.0
(Affiliated to Bharathidasan University)
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Founders

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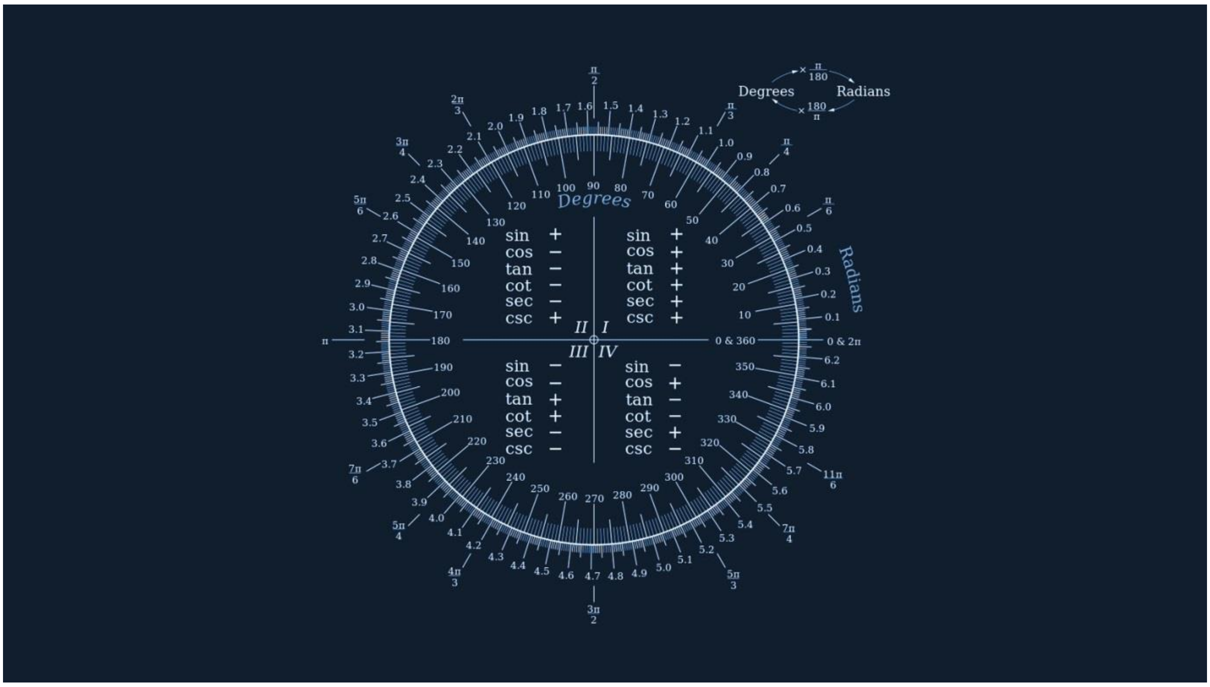
Introduction

This booklet is the brainchild of motivated Mathematics students & Scholars who wish to disseminate mathematical information regarding the reputed Mathematical Institutions, current events, unsolved problems, Millennium prize problems, puzzles, solutions etc.,



About The College

Jamal Mohamed College was founded in 1951, as an affiliated college to the University of Madras and then affiliated to Bharathidasan University, Tiruchirappalli, since its inception in 1982. The College is administrated by the Society of Jamal Mohamed College. It is established in a sprawling land area of 87 acres as a religious minority institution with the primary objective of providing higher education to the downtrodden and socially backward sections of the society in general and Muslim minority in particular. Over the period of 6 decades, the college was able to scale greater and greater heights and rise to the present status as a multi-faculty institution with 11 UG programmes, 11 PG programmes and 2 M.Phil programmes under Government Aided stream and 8 UG programmes, 15 PG programmes, 15 M.Phil and 15 Ph.D programmes under self-financing stream for men by promoting quality and excellence in higher education due to the sustained efforts and dedicated leadership given by the College Management Committee. The college also offers 12 UG, 14 PG programmes, 15 M.Phil and 15 Ph.D programmes exclusively for women as part of women empowerment during the second shift of the college.

About The Department

The Department of Mathematics of Jamal Mohamed College, was started as one of the earliest departments in the year 1951. B.Sc., and M.Sc Mathematics programmes were started in 1957 and 1963 respectively. The Department was elevated into a research department in the year 2002 by offering M.Phil (Full-Time & Part-Time) and Ph.D (Full-Time & Part-Time) programmes. B.Sc Mathematics (Self-Financing Section) was started for women in 2003. M.Sc Mathematics (Self-Financing Section) was started for women and men in 2005 and 2010 respectively.

In the year 2008, the Department was nationally identified as a Department with potential for award of FIST-Grant by the Ministry of Science and Technology of Government of India.

The Department has highly qualified faculty members actively engaged in teaching, research, continuing education programmes and consultancy. In the last 11 years, the members of the Department have written 10 books, 42 research scholars were awarded Ph.D degrees, published more than 482 research papers in National and International journals and presented many research papers in National and International Conferences.

A Matrix Proof of Newton's Identities

*Dan Kalman

Newton's identities relate sums of powers of roots of a polynomial with the coefficients of the polynomial. They are generally encountered in discussions of symmetric functions a polynomial's coefficients are symmetric functions of the roots, as in the sum of the kth powers of those roots.

Newton's identities also have a natural expression in the context of matrix algebra, where the trace of the kth power of a matrix is the sum of the kth powers of the eigenvalues. In this setting, newton's identities can be derived as a simple consequence of the Cayley-Hamilton theorem. Presenting that derivation is the purpose of this note.

There are a variety of derivations for Newton's identities in the literature. Berlekamp's derivation using generating function methods is short and elegant, and mead presents a very interesting argument using a novel notation. In yet another approach, Baker uses differentiation to obtain a nice recursion. Eidswick's derivation uses a related application of logarithmic differentiation. All of these proofs are elementary and understandable, but they involve manipulation or concepts that might make then a bit forbidding to students.

In contrast the proof presented here uses only methods that would be readily accessible to most linear algebra students.

Interstingly, the matrix interpretation of Newton's identities is familiar in the linear algebra literature, providing a means of computing the characteristic polynomial of a matrix in terms of the traces of the powers of the matrix. However, using the matrix setting to derive Newton's identities doesn't seem to be well known.

Let $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_0$ have roots $r_j, j = 1, \ldots, n$. Define

$$s_k \equiv \sum_{j=1}^{n} r_j^k$$

Newton's identities are

$$s_k + a_{n-1}s_{k-1} + \cdots + a_0 s_{k-1} = 0 \quad (k > n)$$

$$s_k + a_{n-1}s_{k-1} + \cdots + a_{n-k+1}s_1 = -k_{a_{n-k}} \quad (1 \le k \le n)$$

Now let C be an $n \times n$ matrix with characteristic polynomial equal to p. For example, C might be

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}$$

Then the roots of p are the eigenvalues of C and more generally, the kth powers of the roots of p are the eigenvalues of C^k. Accordingly, we observe that s_k is the trace of C^k

written (trC^k). Recall that the trace matrix is at once the sum of the eigenvalues and the sum of the diagonal entries. In particular, the trace operation is linear $tr(\alpha A+\beta B)=\alpha tr(A)+\beta tr(B)$

Now for $k>n$, using the race formulation, Newton's identity becomes

$$tr(C^k)+a_{n-1}tr(C^{k-1})+\cdots + a_0 tr(C^{k-n})=0$$

And since the trace function is linear, we can rewrite this as

$$tr(C^k+a_{n-1}C^{k-1}+\cdots+a_0C^{k-n}) = 0, \quad or \quad tr\left(C^{k-n}p(c)\right)=0$$

Thus, the $k>n$ case follows immediately from the Cayley-Hamilton theorem, which says that $p(C)=0$

For $1\le k\le n$, the trace version of Newton's identity is

$$tr(C^k)+a_{n-1}tr(C^{k-1})+\cdots + a_{n-k+1}tr(C)=-ka_{n-k}$$

Which can again be rewritten as

$$tr(C^k+a_{n-1}C^{k-1}+\cdots+a_{n-k+1}C) = -ka_{n-k}$$

For reasons that will be clear later, we modify this slightly, to

$$tr(C^k+a_{n-1}C^{k-1}+\cdots+a_{n-k+1}C + a_{n-1}I) = (n-k)a_{n-k} \quad .... \quad (1)$$

This identity can also be derived from the Cayley-Hamilton theorem, in a slightly different way. As is well known, a real number r is a root of a real polynomial $p(x)$ if and only if $(x-r)$ is a factor of $p(x)$, and the complimentary factor can be determined using synthetic division. This situation can be mimicked exactly using matrices: let $X=xI$, and divide $p(x)$ by $X-C$ using synthetic division. Since $p(C)=0$, the division terminates without remainder, providing the factorization

$$p(X)=(X-C)[X^{n-1} + (C+a_{n-1}I)X^{n-2} + (C^2+a_{n-1}C + a_{n-2}I)X^{n-3}+\cdots + (C^{n-1}+a_{n-1}C^{n-2}+\cdots + a_1I)I]$$

To relate this to equation (1) we will want to introduce the trace operation. Unfortunately, the trace does not relate well to matrix products, so it is necessary to eliminate the factor $(X-C)$ on the right. Fortunately, as long as x is not an eigen value of C, we know that $(xI-C)=(X-c)$ is non-singular, so we can write

$$(X-C)^{-1}p(X)=X^{n-1}+(C+a_{n-1}I)X^{n-2} + (C^2+a_{n-1}C+a_{n-2}I)X^{n-3} + \cdots + (C^{n-1}+a_{n-1}C^{n-2}+\cdots + a_1I)I$$

Taking the trace of each side then leads to

$$tr[(X-C)^{-1}p(X)] = nx^{n-1} + tr(c+a_{n-1}I)x^{n-2}+\cdots + tr(C^{n-1}+a_{n-1}C^{n-2}+\cdots + a_1I) \ldots (2)$$

Because $tr(I) = n$ and $tr(X^k A) = tr(X^k IA) = x^k tr(A)$ for any matrix A.

We will next show that the left side of this equation is none other than $p'(x)$. Then comparing coefficient on either side will complete the proof. Indeed, equating the coefficient of x^{n-k-1} in $p'(x)$ with the corresponding coefficient on the right side of equation (2) gives

$$(n-k)a_{n-k} = tr(C^k + a_{n-1}C^{k-1} + \cdots + a_{n-k+1}C + a_{n-k}I)$$

Which is exactly the same as equation (1) So, consider $A = (X - C)^{-1}p(X)$. Observe that $p(X) = p(xI) = p(x)I$, so we can equally well write $A = p(x)(xI - C)^{-1}$. This shows that

$$tr(A) = p(x)tr(xI - C)^{-1}$$

Now the trace of any matrix is the sum of its eigenvalue (with multiplies as in the characteristic polynomial). And the eigenvalues of $(xI - C)^{-1}$ are simply the fractions $\frac{1}{x-r_1}, \frac{1}{x-r_2}, \ldots, 1/(x - r_n)$. This shows

$$tr(A) = p(x)\left(\frac{1}{x-r_1} + \frac{1}{x-r_2} + \cdots + \frac{1}{x-r_n}\right)$$

Which is immediately recognizable as the derivative $p'(x)$ (using the fact that $p(x) = (x - r_1)(x - r_2) \ldots (x - r_n)$). This completes the proof

Courtesy: Mathematical Associations of America 2000, volume 73, No:4

Archimedes in the 5th Dimension

Dan Kalman

Among the many discoveries of Archimedes, the renowned Greek mathematician (ca 250 BC), one that is particularly striking relates the volumes of a sphere, cylinder and cone. This leads to our familiar formula for the volume of a sphere, $V = (4/3)\pi r^3$ and also revealed to Archimedes the surface area of the sphere. Indeed, Archimedes deduced that the ratio between the volume of a sphere and its circumscribing cylinder is the same as the ratio between the surface areas of these solids. So great was his admiration of this discovery that he had it inscribed on his tombstone.

Archimedes drew on mechanical principles (particularly the law of the lever) to make such discoveries, although he also constructed geometric proofs to substantiate his results. The argument relating the sphere, cone, and cylinder is formulated in terms of equilibria and balancing points. The heart of the argument can be understood a bit more easily in terms of simple geometry: The volume of a cylinder is equal to the sum of the volumes of a sphere and a cone because the same can be said about their cross-sectional slices. Interestingly, this idea can be extended very naturally to five dimensional figures, as we shall see below. First, though, we will take a more detailed look at the situation in our three familiar dimensions.

In three dimensions

Consider a hemisphere, cylinder, and cone, as shown in Figure 1.

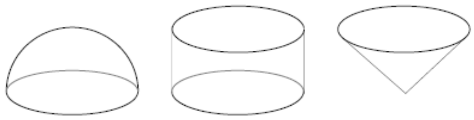


Figure: 1 sphere, cylinder, cone

The dimensions of the three figures line up. That is, they all have equal height *r*, the radius of the hemisphere, and this same *r* is the radius of the circular base of the cylinder and top of the cone. Note that this also implies that there is a 45 degree angle between the central axis of the cone and the lateral surface.

Now consider a cross sectional slice through all three figures, at a common height *h*, as in Figure 2.

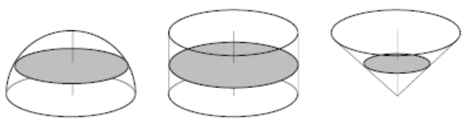


Figure: 2 slice at height h

Each figure has a circular cross section. For the cylinder, the radius of the circle is *r*. For the cone, the radius is equal to *h*. And for the sphere, the radius of the slice is $\sqrt{r^2 - h^2}$ see fig 3

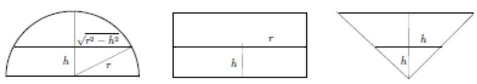


Figure: 3 side view

Accordingly, the areas of these slices are, respectively, $\pi r^2, \pi h^2 \ and \ \pi(r^2 - h^2)$. We see at once that the area of the hemispherical slice plus the area of the conical slice equals the area of the cylindrical slice. Therefore, the volume of the hemisphere plus the volume of the cone equals the volume of the cylinder.

Now we can use known formulas for volumes of cylinders and cones to deduce the volume of the hemisphere. Since the cylinder has volume $\pi r^2 . r = \pi r^3$ and since the cone has just one third of that volume, the hemisphere must have volume $V = (2/3)\pi r^3$. Of course, the full sphere is twice as large, giving the familiar $V = (4/3)\pi r^3$ for the volume of a sphere of radius *r*.

On to dimension 5

What does it mean to talk about volumes in five dimensions? First of all, we think of five dimensional space as made up of points (*x*, *y*, *z*, *w*, *u*) whose coordinates refer to positions along five mutually perpendicular axes. I have never actually *seen* five mutually perpendicular axes; to do so, I would have to exist in five dimensional space. So I have to use my imagination, and take the existence of these axes as a given.

In fact, there are a number of aspects of volumes that will just be assumed, in analogy with the geometry of three dimensions. One of these is the idea of distance or length. The distance from the origin to (*x*, *y*, *z*, *w*, *u*) is given by

$$\sqrt{x^2 + y^2 + z^2 + w^2 + u^2}$$

This follows from the Pythagorean theorem, and some reasonable assumptions about perpendicularity, and can be found in any calculus or linear algebra book.

Continuing, a theory of volume in five dimensions can be developed in perfect analogy with volume in three dimensions. We define a unit cube to be a set of points something like this:

$$0 \leq x \leq 1$$
$$0 \leq y \leq 1$$
$$0 \leq z \leq 1$$
$$0 \leq w \leq 1$$
$$0 \leq u \leq 1$$

and declare that it has a volume of 1. Visualize it as the region of space that has width 1 parallel to each of the five perpendicular axes. We can also think of this as the Cartesian product of five copies of the unit interval [0,1].

We assume that five dimensional volume is additive and translation invariant. That means that if you decompose a figure into pieces, the volume of the whole is the sum of the volumes of the pieces, and that congruent pieces have equal volume. From these assumptions, one deduces that the volume of a product of five intervals is the product of the lengths of the intervals. For example, a five dimensional rectangular prism with dimensions 1 by 2 by 3 by 4 by 5 would have volume 1.2.3.4.5 = 120.

Another important aspect of volumes is also related to Cartesian products. Suppose that (x, y, z) is free to roam throughout a three dimensional region R, and that (w, u) is free to roam about a plane region S. Then all possible points (x, y, z, w, u) with (x, y, z) in R and (w, u) in S populate a region in five dimensional space denoted by $R\times S$. The volume of that region is found by multiplying the volume of R by the volume of S. This is really the same idea as described above for the product of five intervals. It also leads to the familiar formulas for three dimensional volumes of cylinders (area of the base times the height). One has to be a little relaxed with the word *volume* however. If we are talking about a two dimensional object, volume really means area. For a one dimensional object volume is length. This collision of terminology inspires many writers to adopt a neutral term, *measure*, that stands for length, area, and volume depending on the dimensional context. But I will stick with *volume*, trusting you to understand when I really mean area or length.

At last, it is possible to describe the five dimensional version of Archimedes' observation linking the hemisphere, cone, and cylinder. In five dimensions, we consider four figures: a 5D-hemisphere, a 5D-cylinder, a 5D-cone, and a circle-cone. The circle-cone is actually a Cartesian product $R\times S$ where R is a two dimensional filled-in plane circle (hereafter known as a disk), and S is a three dimensional cone. We slice each of the four objects at a single height h along the u axis, and find that the volume of the slice of the 5D-hemisphere is equal to the volume of the slice of the 5D-cylinder + the slice of the 5Dcone – the slice of the circle-cone. Therefore, the five dimensional volume of the full figures made up of all such slices share the corresponding equation: hemisphere = cylinder + cone – circle-cone. As in the three dimensional case, this leads to a formula for the volume of the sphere.

For all of this to make sense, you need to have some idea what I mean by spheres, cones, cylinders, and

slices in five dimensions. Let's begin with the sphere, made up of all the points at distance R or less from the origin. The point (x, y, z, w, u) is in the sphere if and only if $x^2 + y^2 + z^2 + w^2 + u^2 \leq r^2$.
Similarly, a point (x, y, z, w) is in the 4-sphere of radius a if and only if $x^2 + y^2 + z^2 + w^2 \leq a^2$. The four dimensional volume of this four-sphere is given by V = $(1/2)\pi^2 a^4$. This can be derived through an appropriate slicing analysis, but it will be too great a distraction to go into that in detail now.

Based on our understanding of the 5-sphere, what should we mean by a slice? In analogy with the three dimensional case, a slice perpedicular to the u axis is determined by assigning a fixed value to u. For example, set u = 2. The points of the sphere that have the form (x, y, z, w, 2) make up one slice of the sphere. And such a point will satisfy $x^2 + y^2 + z^2 + w^2 \leq r^2 - 4$.

Observe that this slice is a four dimensional object, in fact, a sphere of radius $\sqrt{r^2 - 4}$ If we slice at u = h, we obtain a four dimensional sphere of radius $\sqrt{r^2 - h^2}$ Using the volume formula for the 4-sphere, we obtain V = $(1/2)\pi^2(r^2 - h^2)^2$ as the volume of one slice of the 5-sphere. Actually, we are only looking at slices for positive values of h, so we should really think of these as slices of a hemisphere.

Next consider a cylinder. In three dimensions, a cylinder is a Cartesian product of a disk with a perpendicular line segment. For example, consider the points (x, y, z) for which (x, y) is on or in the unit circle in the plane, and z lies between 0 and 1. That is precisely the description of a cylinder centered on the z axis with radius 1 and height 1. Each cross sectional slice is a disk of radius 1.

The five dimensional analog of this is a Cartesian product of a four dimensional sphere with an interval. For the case at hand, we allow (x, y, z, w) to be any point in the 4-sphere of radius r, and restrict u to lie between 0 and r. This defines our five dimensional cylinder, with equal radius and height r. Slicing this object at u = h produces a 4-sphere of radius r. The four dimensional volume of the slice is $(1/2)\pi^2 r^4$.

The five dimensional cone is just a little more involved. Again begin with the three dimensional version. One way to characterize that cone is by describing its slices. We know that each slice is disk, and that radii of these disks increase linearly from 0 at the vertex to whatever radius we have specified for the top of the cone. If the height and top radius are both r, then the radius of each slice is equal to the height of that slice above the vertex. That is, the slice at height h will be a disk of radius h. Now we can extend this idea by analogy to five dimensions. We want the slice at u = h to be a 4-sphere of radius h.

In symbols, u = h implies $x^2 + y^2 + z^2 + w^2 \leq h^2$. That leads to the inequality $x^2 + y^2 + z^2 + w^2 \leq u^2$ as the definition of our cone. This agrees with other conceptualizations of the cone. For example, for any point P on the cone, the line from the origin to P makes a constant angle with the u axis.

In any event, with this understanding of a cone, we see that the slice at $u = h$ is a 4-sphere of radius h. The four dimensional volume of this slice is $(1/2)\,\pi^2 h^4$.

Finally we come to the circle-cone. This is defined as a Cartesian product. Let (x, y) be any point of a disk of radius r. Thus, $x^2 + y^2 \leq r$. Let (z, w, u) be any point of a cone centered on the u-axis with apex at the origin, and whose height and top radius each equal r. Then $z^2 + w^2 \leq u^2$. The circle cone is made up of the points (x, y, z, w, u) satisfying both conditions.

Now what is the slice for u equal to a fixed value h? We still can have any (x, y) in the disk of radius r. But now we have (z, w, u) with $z^2 + w^2 \leq h^2$ because $u = h$. This shows that the slice

is a Cartesian product of two planar disks, one with radius r and the other with radius h. The four dimensional volume of the slice is therefore $\pi r^2 . \pi h^2 = \pi^2 r^2 h^2$.

At this point, we are in a position to compare the corresponding slices of the hemisphere, cylinder, cone, and circle cone. To simplify the comparison, the earlier results are combined in the following table:

figure	Volume of slice
Hemisphere	$\frac{1}{2}\pi^2(r^2 - h^2)^2$
Cylinder	$\frac{1}{2}\pi^2 r^4$
Cone	$\frac{1}{2}\pi^2 h^4$
Circle-cone	$\pi^2 r^2 h^2$

It is evident that the volumes of these slices obey the Archimedes-like equation

Hemisphere = Cylinder + Cone – Circle-Cone.

Consequently, the five dimensional volumes of these solids must be related in the same way. Now the volumes of the cylinder, cone, and circle-cone are easy to find. For the cylinder, volume is base times height, where the base is a 4D sphere of radius r, and the height is also r. This gives $V_{cyl} = (1/2)\pi^2 r^5$. Next, we need the volume of a cone in five dimensions. Recall that in three dimensions, the volume for a cone is 1/3 base×height. It turns out that in four dimensions the formula is 1/4 base × height and in five dimensions it is 1/5 base × height. So the volume of our cone is 1/5 the volume of the cylinder, and hence, $V_{cone} = (1/10)\pi^2 r^5$. As the circle-cone is a Cartesian product, it's volume is found by multiplying the volume of the three dimensional cone $(1/3)\pi r^3$ by the area of the circle (πr^2). Thus, $V_{cir-cone} = (1/3)\pi^2 r^5$.

Putting all of these results together, we find

$$V_{hemisphere} = V_{cyl} + V_{cone} - V_{cir-cone}$$

$$= \pi^2 r^5 \left(\frac{1}{2} + \frac{1}{10} - \frac{1}{3}\right)$$

$$= \frac{4}{15}\pi^2 r^5$$

This gives the volume for five dimensional sphere as $V = (8/15)\pi^2 r^5$

This is a result and a derivation that Archimedes would have appreciated. To be sure, a few of the missing details should be checked. These include the formula for the volume of a 4-sphere, as well as the pattern relating volumes of cylinders and cones in various dimensions. Both are readily verified using a slicing analysis and simple integration. You should be able to work these out on your own, now that you are familiar with higher dimensional slicing.

It is fun to see how much you can figure out about geometry in higher dimensions, and it is surprising how much you can work out in great detail. An interesting topic for further exploration is the volume of spheres in n dimensions. In fact, there are nice formulas for spheres in every dimension, but the even and odd dimensions follow different patterns. Here is one nice tidbit: for even dimension $2n$ the volume of the unit sphere is $\pi^n/n!$. That means if you add up the volumes of all the even dimensional unit spheres, you get e^π. For more information on this topic, try doing an internet search on the volume of an n-sphere. Similarly, you can find many interesting references related to Archimedes and the volume of the sphere by searching for *Archimedes sphere cone cylinder*. For a more comprehensive and conventional source on the work of Archimedes, see Sherman Stein's 1999 MAA book *Archimedes: What Did He Do Besides Cry Eureka*?

Courtesy: Math Horizons

Mathematics and Geometric Ornamentation in the Medieval Islamic World

Jan P. Hogendijk

Introduction

Many medieval Islamic mosques and palaces are adorned with highly intricate geometric ornaments. These decorations have inspired modern artists and art historians, and they have been discussed in connection with modern mathematical concepts such as crystallographic groups and aperiodic tilings. The Islamic ornamental patterns can certainly be used to illustrate such modern notions.

Medieval Islamic civilization has also left us an impressive written heritage in mathematics. Hundreds of Arabic and Persian mathematical manuscripts have been preserved in libraries in different parts of the world. These manuscripts include Arabic translations of the main works of ancient Greek geometry such as the *Elements* of Euclid (ca. 300 BC) and the *Conics* of Apollonius (ca. 200 BC), as well as texts by medieval authors between the 8th and 17th centuries, with different religious and national backgrounds. In what follows reference will be made to 'Islamic' authors and 'Islamic' texts but the word 'Islamic' will have a cultural meaning only. Most 'Islamic' mathematical texts were not related to the religion of Islam and, although the majority of 'Islamic' authors were Muslims, substantial contributions were made by Christians, Jews and authors with other religious backgrounds who lived in the Islamic world.

Many Islamic texts on geometry are related to spherical trigonometry and astronomy, and most Islamic scholars who studied the *Elements* of Euclid were studying in order to become astronomers and possibly astrologers. Yet there are also Islamic works on geometrical subjects unrelated to astronomy. In almost all medieval Islamic geometrical texts that have been published thus far, one does not find the slightest reference to decorative ornaments. This may be surprising because the authors of these texts lived in the main Islamic centres of civilization and may have seen geometric ornaments frequently.

The Topkapı Scroll

The craftsmen themselves seem to have left us with very few documents about their activities in the field of geometric ornamentation. The most important published example is the socalled Topkapı Scroll, which is now preserved in the Topkapı Palace in Istanbul and which has appeared in a magnificent volume. This 29.5m long and 33 cm wide paper scroll is undated and may have been compiled in North-western Iran in the 16th century but the dating is uncertain. The scroll consists of diagrams without explanatory text. Many of these diagrams are related to calligraphy or muqarnas and therefore do not concern us here. Some of the diagrams concern plane tilings. I have selected one non-trivial example in order to draw attention to the characteristic (and frustrating) problems of interpretation. The drawing on the scroll [14, p. 300] consists of red, black and orange lines, which are indicated by bold, thin and broken lines respectively in Figure 1 (for a photo of the manuscript drawing see also). The broken lines in Figure 1 define a set of five tiles, called *gireh*-tiles in modern research literature from the Persian word *g¯ıreh*, which means knot. The thin lines form a decorative pattern which can be obtained by bisecting the sides of the gireh-tiles and drawing suitable straight line segments through the bisecting points. It is likely that the pattern was designed this way but one cannot be sure because the scroll does not contain any explanatory text. The gireh tiles of Figure 1 have drawn recent attention because they can be used to define aperiodic tilings. In the absence of textual evidence, it is impossible to say whether the craftsmen had an intuitive notion of aperiodicity.

An anonymous Persian treatise

One would like to have a medieval Islamic treatise, written by a craftsman, in which the design and construction of ornaments is clearly explained. Such a treatise has not been found and, thus far, only a single manuscript has been discovered in which diagrams on geometrical ornaments are accompanied by textual explanations. In this section we will discuss what this manuscript can tell us about the main question at the beginning of the paper. The manuscript is a rather chaotic collection of 40 pages of Persian text and drawings. The text consists of small paragraphs which are written close to the drawing to which they refer and, although the texts and drawings appear in a disorganised order and may not be the work of a single author, the collection will be considered as one treatise.3 It may have been compiled in the 16th century, although some of the material must be older as we shall see.

The treatise belongs to a manuscript volume of approximately 400 pages. Some of the other texts in the manuscript volume are standard mathematical works such as an Arabic translation of a small part of Euclid's *Elements*. But the treatise itself does not resemble a usual work by a mathematician or astronomer in the Islamic tradition. It is assumed here that the treatise is the work of one or more craftsmen because it agrees with most of what Abu'l-Wafˉa' says about their methodology.

The treatise provides much additional information on the working methods of the craftsmen and it also shows that they were really involved with the design and construction of geometrical ornaments. In order to illustrate these points, the following four examples 4.1 through 4.4 have been selected from the treatise.

The treatise contains many approximation constructions, including a series of ruler-and-compass constructions of a regular pentagon by means of a single compass-opening. In these constructions, the compass-opening is assumed to be either the side of the required regular pentagon or the diagonal, the altitude or the radius of the circumscribing circle. Here is one such construction with a paraphrasing of the manuscript text [12, 184b]. Figure 2 is a transcription of the figure in the manuscript, in which the labels (the Arabic letters *alif*, *bˉa'*, . . .) are rendered as *A*, *B*, . . . , and Hindu-Arabic number symbols are represented by their modern equivalents. The Persian text says:

On the construction of *gonia* 5 by means of the compass-opening of the radius, from *gonia* 6. On line *AG* describe semicircle *ADG* with centre *B*. Then make point *A* the centre and describe arc *BE*. Then make point *G* the centre and on the circumference of the arc find point *D* and draw line *AD* to meet arc *EB* at point *Z*.

Draw line *GZ* to meet the circumference of the arc at point *H*. Join lines *AH*,*GH*.4 Each of the triangles *AZH*,*GZD* is *gonia 5*, and the original triangle *ADG* was *gonia* 6, . . .

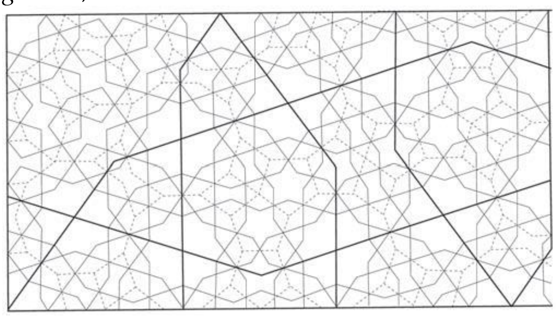


Figure: 1 Drawing by Dr Steven Wepster

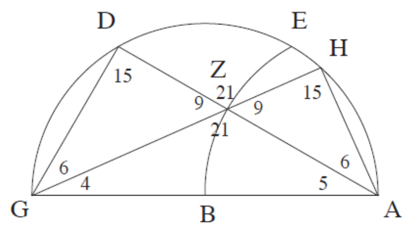


Figure 2.

Points *A*, *E*, *D* and *G* are four angular points of a regular hexagon and *DH* is the side of the regular pentagon inscribed in the same circle. The construction is a good approximation5 but it is not exact so Abu'l-Wafˉa' would not have approved it. In Chapters 3 and 4 of his booklet, Abu'l-

Waf¯a’ provided exact constructions of the regular pentagon using a fixed compass-opening. The *gonia* is mentioned by Abu’l- Waf¯a’ as an instrument used by craftsmen. From the Persian treatise we infer that *gonia n* is a set square with angles 90^0, $\frac{180^0}{n}$ and $90-\frac{180^0}{n}$ In Figure 2, angles are expressed in units such that 15 units are a right angle. In the Islamic tradition, the division of the right angle into 90 degrees, subdivided sexagesimally, was only used in mathematical astronomy and mathematical geography.

Abu’l-Waf¯a’ says that the craftsmen are interested in cut-and paste constructions, and the Persian treatise contains many such constructions. Some of these are explained by one or more paragraphs of text but the following example is presented without accompanying text.

Figure 3 displays a regular hexagon and an isosceles triangle, dissected into pieces such that both figures can be composed from these pieces. Figure 3 is derived from the manuscript [12, 197a] with the difference here that the isosceles triangle has been arbitrarily assumed to be equilateral and the figure has been drawn in a mathematically correct way. In the manuscript, the pieces are indicated by numbers (as in Figure 3) so the correspondence is clear. Since there is no text in the manuscript, the reader does not have a hint of how exactly the pieces have to be cut. Readers are invited to work out the details for themselves. After this exercise, they will probably be convinced that the manuscript was intended to be used under the guidance of a competent teacher who could provide further information. It should be noted that the pieces no. 1 and 2 in the manuscript are drawn in such a way that no. 1 is wider than no. 2. This may happen if the vertex angle of the isosceles triangle is less than 54^0; Figure 4 has been drawn for a vertex angle of $\frac{360^0}{7}$. It is tempting to assume that the craftsmen had a general dissection of an isosceles (rather than an equilateral) triangle in mind but because there is no accompanying text, one cannot be sure. The construction is mathematically correct but there are also approximate cut-and-paste constructions in the Persian treatise.

It is not necessary to assume that the fancy cut-and-paste construction of Figures 3 and 4 was used in practice. Just like European arithmetic teachers in later centuries, Islamic craftsmen may have challenged one another with problems which surpassed the requirements of their routine work.

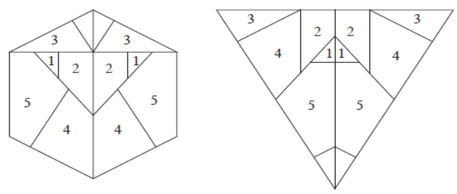


Figure 3.

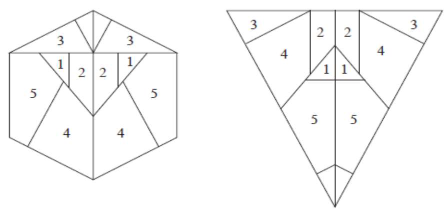


Figure 4.

The many drawings of geometric ornaments in the Persian treatise show that its authors were deeply involved with the design and construction of ornamental patterns. This example is also found on a real building, namely the North Cupola of the Friday Mosque in Isfahan, which was built in the late 11th century.

Make angle *BAG* three sevenths of a right angle. Bisect *AG* at point *D*. Cut off *BE* equal to *AD*. Produce line *EZ* parallel to *AG*. Draw line *T I* parallel to *BE*, bisect *TE* at point *H* and make *T I* equal to *TH*. Extend *EI* until it intersects *AB* at point *K*. Produce *KL* parallel to *BE*. With centre *Z* draw circular arc *KMN* in such a way that its part *KM* is equal to *MN*. On line *AF* take point *S* and that is the centre of a heptagon. Complete the construction, if God Most High wants.

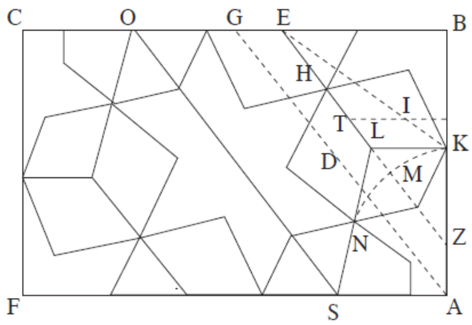


Figure 5.

Or construct angle *ELN* equal to angle *ELK* and by means of line *LN* find the centre *S*. Or cut off *EO* equal to *EL*, so that *O* is the centre of a heptagon. And make line *OS* parallel to *GA* and equal to *AG*. Then point *S* is the centre of another heptagon. Or else let *GO* be equal to *AS*. God knows best.

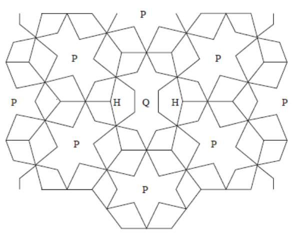


Figure 6.

The text does not inform the reader what should be done with the completed figure. Apparently the rectangular figure in the manuscript and its mirror image should be repeated as suggested by Figure 6. Thus one obtains the pattern in the north cupola of the Friday Mosque.

The pattern can be linked to *gireh* tiles such as in Figure 1 above. These *gireh* tiles are not mentioned explicitly in the Persian treatise; all information in the treatise about Figure 5 is contained in the passages quoted above. Let $\alpha = \frac{1}{7} \times 180^0$ and take as gireh tiles two types of equilateral hexagons with equal sides (thin lines in figure 6), of type *P* with angles 4α, 5α, 5α, 4α, 5α, 5α and of type *Q* with angles 4α, 4α, 6α, 4α, 4α, 6α. Now draw suitable lines through the midpoints of the sides, in such a way that the "stars" inscribed in *P* and *Q* emerge, with angles 2α at the midpoints of the sides of the gireh tiles. The heptagons *H* in Figure 6 are regular. Patterns with regular heptagons are rarely found on Islamic buildings so the pattern in the manuscript and on the North Cupola probably go back to the same designer or designers. The pattern on the North Cupola of the Friday Mosque consists of the thick lines in Figure 6 with some additional embellishments but without the *gireh* tiles in Figure 6.

The fourth and final example from the Persian treatise will reveal some information about the relationship between craftsmen and Islamic mathematician-astronomers who had been trained in Greek mathematics. As an introduction, consider a pattern from the Hakim Mosque in Isfahan (Figure 7). The pattern is inspired by a division of a big square into a small square and four kites.10 Two of the angles of each of the kites are right angles.

Figure 8 is a partial transcription of a figure in the Persian treatise [12, 189b] but the labels and broken lines are additions.11 The figure displays a big square with side *ZP*, subdivided into a small square with side *RQ* and four big kites such as *EQTZ* and *RTPU*, each with two right angles and with pairwise equal sides (*QE* = *EZ*, *QT* = *TZ*, *RT* = *TP*, *RU* = *UP*). Note that the four longer diagonals of the

Figure 7.

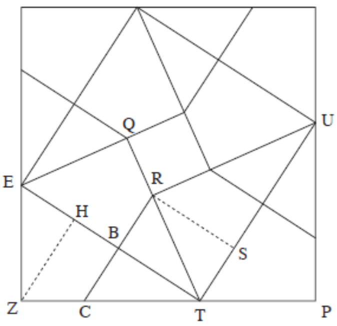


Figure 8.

big kites also form a square with side *ET*, which will be called the intermediate square. In the special case of Figure 8, the side *QR* of the small square is supposed to be equal to the distance *RB* between each angular point of the small square and the closest side of the intermediate square. Then each big kite such as *EQTZ* can be divided into two right-angled triangles *BRT*, *BCT* and two small kites such as *EQRB*, *EBCZ* with two right angles and pairwise equal sides (*EQ* = *EB*, *RQ* = *RB*, *EB* = *EZ*, *CB* = *CZ*). Thus we have four big kites and eight small kites and, for easy reference, the resulting division of the big square will be called the twelve kite pattern.

Almost a quarter of the Persian treatise is somehow devoted to the twelve kite pattern. If we draw perpendiculars *ZH* and *RS* to *ET* and *TU* respectively, *ZH* = *RS* = *BT*. The two sides *EZ* and *EB* of the small kite *EBCZ* are also equal, so in the right-angled triangle *EZT* we have *ZH* + *EZ* = *ET*. The twelve kite pattern can be constructed if a right-angled triangle (such as *EZT*) can be found with the property that the altitude (*ZH*) plus the smallest side (*ZE*) is equal to the hypotenuse (*ET*). The text states that "Ibn-e Heitham" wrote a treatise on this triangle and constructed it by means of two conic sections, namely "a parabola and a hyperbola". No further details are given and no conic section is drawn anywhere in the Persian treatise. But the text contains a series of approximation constructions of the twelve kite pattern, such as the following [12, 189b] (Figure 9). The text reads:

Line *AD* is the diagonal of a square. The magnitudes of *AB*, *BG* are equal and *AD* is equal to *AB*. Find point *E* on the rectilinear extension of line *GD*. Then each of *EZ*, *ZH* is equal to *AG*. Join

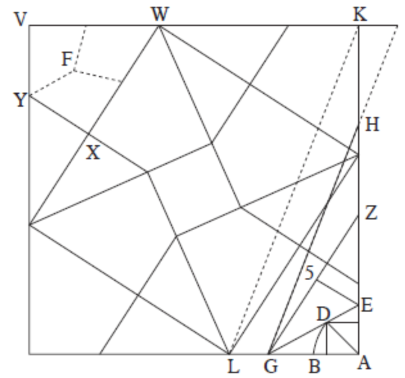


Figure 9.

line *GH* and through point *K* draw line *KL* parallel to *GH*. Find point *L*; the desired point has now been obtained.

The approximation is sufficiently close for all practical purposes: if the side of the square is 1 metre, the difference between the correct and approximate positions of *L* is only a few millimetres.12 It does not follow that the approximation presupposes a deep mathematical knowledge. In the figure in the manuscript, the eight small kites are all subdivided into three even smaller kites with pairwise equal sides and at most one right angle. In Figure 9 the subdivision is indicated by broken lines in only one kite *VWXY* (labels by the author) in the upper left corner. One may guess that *FV* = 12 *VW* and note that *F* is located on the bisector of angle *WVY*. The first step of the approximation boils down to the construction of a triangle *ADG* similar to *VFW*.

Mathematicians on the twelve kite pattern

The reference to "Ibn e-Heitham" in the Persian treatise shows that the twelve kite pattern was also studied by mathematician-astronomers. We will now discuss what is known about these studies because they will give us some further hints about the interactions between mathematician-astronomers and craftsmen. "Ibn e-Heitham" is a Persian form of Ibn al-Haytham (ca. 965–1041), a well-known Islamic mathematician- astronomer who was interested in conic sections. His treatise on the twelve kite pattern has not been found but one of the extant works of the famous mathematician astronomer and poet *c*Umar Khayy¯am (1048–1131) is also of interest here. The work is written in Arabic and entitled "treatise on the division of a quadrant". It begins in the following uninspiring way (Figure 10, [10, 73]): "We wish to divide the quadrant *AB* of the circle *ABGD* into two parts at a point such as *Z* and to draw a perpendicular *ZH* onto the diameter *BD* in such a way that the ratio of *AE* to *ZH* is equal to the ratio of *EH* to *HB*, where *E* is the centre of the circle and *AE* is the radius." Khayy¯am does not give the slightest indication of the origin or relevance of this problem. He draws the tangent to the circle at *Z*, which intersects *BE* extended at *T* and he shows that in the right angled triangle *EZT*, the sum of the altitude *ZH* plus the shortest side *ZE* is equal to the hypotenuse *ET*.13 Thus the problem is inspired by the twelve kite pattern but Khayy¯am does not mention the relationship with this pattern or with geometric ornamentation in general. In a new figure (not rendered here), Khayy¯am puts, in the notation of Figure 10, *EH* = 10 and *ZH* = x, so $ZE = \sqrt{100 + x^2}$ and by similar triangles $HT = \frac{x^2}{10}$. He then shows that the property *ZH* + *EZ* = *ET* boils down to the cubic equation $x^3 + 200x = 20x^2 + 2000$, or in a literal translation of his words: “a cube and two hundred things are equal to twenty squares plus two thousand in number” [10, 78]. He then proceeds to construct a line segment with length equal to the (positive) root x of this equation by the intersection of a circle and a hyperbola. An anonymous appendix [10, 91] to Khayy¯am’s text contains a direct construction of point Z in Figure 10 as a point of intersection of the circle and the hyperbola through point B whose asymptotes are the diameter AEG and the tangent GM (broken lines in Figure 10). None of this was relevant to a craftsman who wanted to draw the twelve kite pattern and Khayy¯am declares that numerical solutions of the cubic equation could not be found. In order to find a numerical approximation of arc ZB, Khayy¯am rephrases the problem about the quadrant in trigonometrical form as follows: to find an arc such that “the ratio of the radius of the circle to the sine of the arc is equal to the radius of the cosine to the versed sine”. In modern terms, if $\alpha = \angle ZET$ and the radius is 1, the ratio $AE : ZH = EH : BH$ is equivalent to $1 : \sin\alpha = \cos\alpha : (1 - \cos\alpha)$.

Khayy¯am says that this problem can be solved by trial and error using trigonometrical tables and that he found in this way α ≈ 57o, and if AE = 60 then ZH ≈ 50, EH ≈ 3223 and BH ≈ 2713 . He also says that one can solve the problem more accurately. Using the trigonometrical tables that were available in his time, he could have computed the required arc in degrees and minutes by linear interpolation.14 This information on sexagesimal degrees and minutes may not have been of much use to craftsmen as we have already seen in 4.2 above. We may also compare with a reference by the Iranian mathematician and astronomer Al-B¯ır¯un¯ı (976-1043) in a work on the qibla (direction of prayer towards Mecca). Al-B¯ır¯un¯ı computes the qibla at Ghazni (Afghanistan) by trigonometrical methods as 70 degrees and 47 minutes west of the south point on the local horizon. He then adds a ruler-and-compass approximation construction for “builders and craftsmen”, who “are not guided by degrees and minutes” ([4, 286], compare [3, 255–256]).

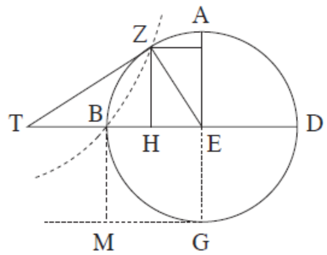


Figure 10.

Conclusion

We now return to the main question in the introduction to this paper. Because the evidence is so scarce, it is not clear to what extent we are able to generalise the information which we can obtain from the available manuscript sources. But if this can be done, the following may be suggested about the main differences between Islamic craftsmen who designed and constructed ornaments and Islamic mathematician-astronomers who were trained in Greek geometry: – Mathematician-astronomers worked with geometric proofs in the style of Euclid’s *Elements*. Craftsmen were familiar with the Euclidean way to draw figures, using letters as labels of points (but also the number 5 in Figure 9 above). Craftsmen did not use geometric proofs and they had not been trained in the methods of Euclid's *Elements*. – Texts written by mathematician-astronomers usually contain sufficient explanation to understand the mathematics. An oral explanation is not absolutely necessary. Texts and diagrams by craftsmen are often ambiguous and oral explanations were essential.

Mathematician astronomers distinguished between exact and approximate geometrical constructions. Craftsmen did not distinguish between these constructions if the result was acceptable from a practical point of view. – Craftsmen used some geometrical instruments not found in the theoretical works of Greek geometry, such as a setsquare and a compass with fixed opening. The following relationship between craftsmen and mathematicians may be suggested. Mathematicians such as Ibn al- Haytham and *c*Umar Khayy¯am may have regarded the designs of craftsmen as a hunting ground for interesting mathematical problems. Thus the twelve kite pattern inspired constructions by means of conic sections, as in Figure 10 above. These constructions were a favourite research topic in the 10th and 11th century among Islamic mathematicians who had studied the *Conics* of Apollonius (ca. 200 BC). However, Khayy¯am did not reveal that his geometric construction problem was inspired by a decorative ornament.15 Other Islamic geometric problems may also have a hitherto unidentified historical context related to ornaments. The craftsmen knew that the mathematicians had worked on some problems related to ornamentation and they regarded the solutions with respect, even though they probably did not understand the details and technicalities. The Persian treatise states [12, 185a] that the construction of a right-angled triangle such as *EZT* in Figure 8 "falls outside the *Elements* of Euclid" and requires the "science of conic sections". No drawing of a conic section occurs anywhere in the Persian treatise. Of course we cannot exclude the possibility that a few mathematicians were also involved in the design and construction of geometric ornaments. The heptagonal pattern in Figure 6 is explained in our treatise in the language of the craftsmen but since *c*Umar Khayy¯am lived in Isfahan at the time that the North Cupola was built, it is possible that he was somehow involved in the design. That a combination of mathematical learning and manual skill was possible in Islamic civilization is shown by the case of Ab¯u H. ¯amid al-Khujand¯ı (ca. 980), who was trained in Greek geometry and astronomy, and who authored a number of geometrical and astronomical works as well as being a superb metalworker. The source materials that we have discussed in this paper give a fascinating glimpse into a design tradition about which little is known. Our knowledge is based to a large extent on one single Persian manuscript which is now preserved in Paris. It is likely that a systematic search in manuscript libraries in the Islamic world will produce many more relevant documents and lead to a significant increase in our insight into the working methods of the medieval Islamic craftsmen…

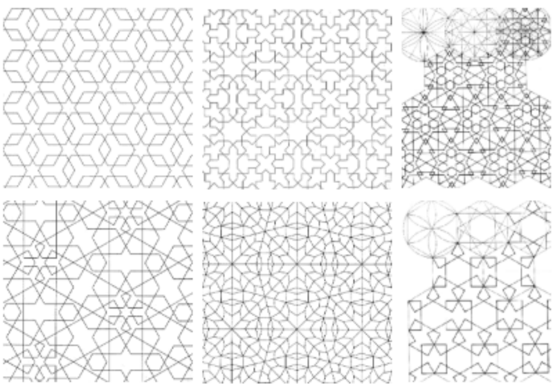
Courtesy: EMS Newsletter December 2012

Islam and Mathematics

The Islamic Empire established across Persia, the Middle East, Central Asia, North Africa, Iberia and parts of India from the 8th Century onwards made significant contributions towards mathematics. They were able to draw on and fuse together the mathematical developments of both Greece and India.

One consequence of the Islamic prohibition on depicting the human form was the extensive use of complex geometric patterns to decorate their buildings, raising mathematics to the form of an art. In fact, over time, Muslim artists discovered all the different forms of symmetry that can be depicted on a 2-dimensional surface.

The Qu'ran itself encouraged the accumulation of knowledge, and a Golden Age of Islamic science and mathematics flourished throughout the medieval period from the 9th to 15th Centuries. The House of Wisdom was set up in Baghdad around 810, and work started almost immediately on translating the major Greek and Indian mathematical and astronomy works into Arabic.



Some examples of the complex symmetries used in Islamic temple decoration

The outstanding Persian mathematician Muhammad Al-Khwarizmi was an early Director of the House of Wisdom in the 9th Century, and one of the greatest of early Muslim mathematicians. Perhaps Al-Khwarizmi's most important contribution to mathematics was his strong advocacy of the Hindu numerical system (1 - 9 and 0), which he recognized as having the power and efficiency needed to revolutionize Islamic (and, later, Western) mathematics, and which was soon adopted by the entire Islamic world, and later by Europe as well.

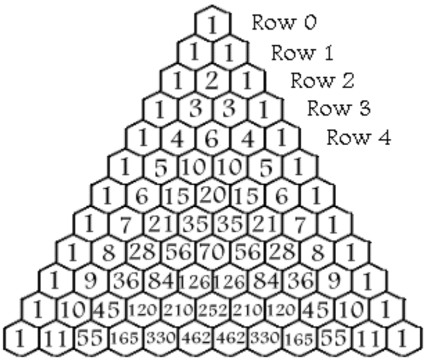
Al-Khwarizmi's other important contribution was algebra, and he introduced the fundamental algebraic methods of "reduction" and "balancing" and provided an exhaustive account of solving polynomial equations up to the second degree. In this way, he helped create the powerful abstract mathematical language still used across the world today, and allowed a much more general way of analyzing problems other than just the specific problems previously considered by the Indians and Chinese.

The 10th Century Persian mathematician Muhammad Al-Karaji worked to extend algebra still further, freeing it from its geometrical heritage, and introduced the theory of algebraic calculus. Al-Karaji was the first to use the method of proof by mathematical induction to prove his results, by proving that the first statement in an infinite sequence of statements is true, and then proving that, if any one statement in the sequence is true, then so is the next one.

The Binomial Theorem can be stated as:

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2}a^{n-2}b^2 + \ldots + b^n$$

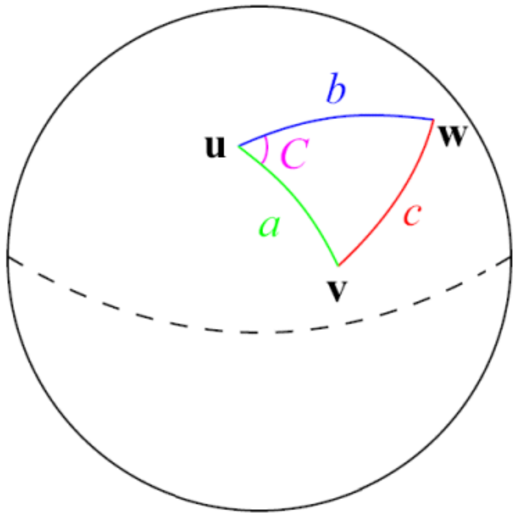
The co-efficients generated by expanding binomials of the form $(a+b)^n$ can be shown in the form of a symmetrical triangle:



Among other things, Al-Karaji used mathematical induction to prove the binomial theorem. A binomial is a simple type of algebraic expression which has just two terms which are operated on only by addition, subtraction, multiplication and positive whole-number exponents, such as $(x + y)^2$. The co-efficients needed when a binomial is expanded form a symmetrical triangle, usually referred to as Pascal's Triangle after the 17th Century French mathematician Blaise Pascal, although many other mathematicians had studied it centuries before him in India, Persia, China and Italy, including Al-Karaji.

Some hundred years after Al-Karaji, Omar Khayyam (perhaps better known as a poet and the writer of the "Rubaiyat", but an important mathematician and astronomer in his own right) generalized Indian methods for extracting square and cube roots to include fourth, fifth and higher roots in the early 12th Century. He carried out a systematic analysis of cubic problems, revealing there were actually several different sorts of cubic equations. Although he did in fact succeed in solving cubic equations, and although he is usually credited with identifying the foundations of algebraic geometry, he was held back from further advances by his inability to separate the algebra from the geometry, and a purely algebraic method for the solution of cubic equations had to wait another 500 years and the Italian mathematicians del Ferro and Tartaglia.

The 13th Century Persian astronomer, scientist and mathematician Nasir Al-Din Al-Tusi was perhaps the first to treat trigonometry as a separate mathematical discipline, distinct from astronomy. Building on earlier work by Greek mathematicians such as Menelaus of Alexandria and Indian work on the sine function, he gave the first extensive exposition of spherical trigonometry, including listing the six distinct cases of a right triangle in spherical trigonometry. One of his major mathematical contributions was the formulation of the famous law of sines for plane triangles, $a/(\sin A) = b/(\sin B) = c/(\sin C)$, although the sine law for spherical triangles had been discovered earlier by the 10th Century Persians Abul Wafa Buzjani and Abu Nasr Mansur.



Other medieval Muslim mathematicians worthy of note include:

The 9th Century Arab Thabit ibn Qurra, who developed a general formula by which amicable numbers could be derived, re-discovered much later by both Fermat and Descartes(amicable numbers are pairs of numbers for which the sum of the divisors of one number equals the other number, e.g. the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220);

The 10th Century Arab mathematician Abul Hasan al-Uqlidisi, who wrote the earliest surviving text showing the positional use of Arabic numerals, and particularly the use of decimals instead of fractions (e.g. 7.375 insead of 7⅜);

The 10th Century Arab geometer Ibrahim ibn Sinan, who continued Archimedes' investigations of areas and volumes, as well as on tangents of a circle;

The 11th Century Persian Ibn al-Haytham (also known as Alhazen), who, in addition to his groundbreaking work on optics and physics, established the beginnings of the link between algebra and geometry, and devised what is now known as "Alhazen's problem" (he was the first mathematician to derive the formula for the sum of the fourth powers, using a method that is readily generalizable); and

The 13th Century Persian Kamal al-Din al-Farisi, who applied the theory of conic sections to solve optical problems, as well as pursuing work in number theory such as on amicable numbers, factorization and combinatorial methods;

The 13th Century Moroccan Ibn al-Banna al-Marrakushi, whose works included topics such as computing square roots and the theory of continued fractions, as well as the discovery of the first new pair of amicable numbers since ancient times (17,296 and 18,416, later re-discovered by Fermat) and the the first use of algebraic notation since Brahmagupta.

With the stifling influence of the Turkish Ottoman Empire from the 14th or 15th Century onwards, Islamic mathematics stagnated, and further developments moved to Europe.

Abel Prize

The Niels Henrik Abel Memorial Fund was established on 1 January 2002, to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize amount is 6 million NOK (about 750,000 Euro) and was awarded for the first time on 3 June 2003.

Russian mathematician receives the 2014 Abel Prize

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2014 to Yakov G. Sinai (78) of Princeton University, USA, and the Landau Institute for Theoretical Physics, Russian Academy of Sciences, "*for his fundamental contributions to dynamical systems, ergodic theory, and mathematical physics*". The President of the Norwegian Academy of Science and Letters, Nils Chr. Stenseth, announced the winner of the 2014 Abel Prize at the Academy in Oslo today, 26 March. Yakov G. Sinai will receive the Abel Prize from His Royal Highness The Crown Prince at an award ceremony in Oslo on 20 May.

The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of NOK 6,000,000 (about EUR 750,000 or USD 1 million).

Yakov Sinai is one of the most influential mathematicians of the twentieth century. He has achieved numerous groundbreaking results in the theory of dynamical systems, in mathematical physics and in probability theory. Many mathematical results are named after him, including Kolmogorov–Sinai entropy, Sinai's billiards, Sinai's random walk, Sinai-Ruelle-Bowen measures, and Pirogov-Sinai theory.

Sinai is highly respected in both physics and mathematics communities as the major architect of the most bridges connecting the world of deterministic (dynamical) systems with the world of probabilistic (stochastic) systems. During the past half-century Yakov Sinai has written more than 250 research papers and a number of books. He has supervised more than 50 Ph.D.-students.

Yakov Sinai has trained and influenced a generation of leading specialists in his research fields. Much of his research has become a standard toolbox for mathematical physicists. The Abel Committee says, "His works had and continue to have a broad and profound impact on mathematics and physics, as well as on the ever-fruitful interaction between these two fields."

Yakov G. Sinai has received many distinguished international awards. In 2013 he was awarded the Leroy P. Steele Prize for Lifetime Achievement from the American Mathematical Society. Other awards include the Wolf Prize in Mathematics (1997), the Nemmers Prize in Mathematics (2002), the Henri Poincaré Prize from the International Association of Mathematical Physics (2009) and the Dobrushin International Prize from the Institute of Information Transmission of the Russian Academy of Sciences (2009).

Many mathematical societies and academies have elected Sinai to membership or honorary membership: the American Academy of Arts and Sciences (1983), the Russian Academy of Sciences (1991), the London Mathematical Society (1992), the Hungarian Academy of Sciences (1993), the United States National Academy of Sciences (1999), the Brazilian Academy of Sciences (2000), the Academia Europaea (2008), the Polish Academy of Sciences (2009) and the Royal Society of London (2009).

The Abel Prize is awarded by the Norwegian Academy of Science and Letters. The choice of the Abel Laureate is based on the recommendation of the Abel Committee, which is composed of five internationally recognized mathematicians. The Abel Prize and associated events are funded by the Norwegian Government.

Fields Medal

The **Fields Medal**, officially known as **International Medal for Outstanding Discoveries in Mathematics**, is a prize awarded to two, three, or four mathematicians not over 40 years of age at each International Congress of the International Mathematical Union (IMU), a meeting that takes place every four years. The Fields Medal is often viewed as the greatest honour a mathematician can receive.[1][2] The Fields Medal and the Abel Prize have often been described as the "mathematician's Nobel Prize".

The prize comes with a monetary award, which since 2006 is $15,000 (in Canadian dollars, roughly US $13,500[3]).[4][5] The colloquial name is in honour of Canadian mathematician John Charles Fields.[6] Fields was instrumental in establishing the award, designing the medal itself, and funding the monetary component.[6]

The medal was first awarded in 1936 to Finnish mathematician Lars Ahlfors and American mathematician Jesse Douglas, and it has been awarded every four years since 1950. Its purpose is to give recognition and support to younger mathematical researchers who have made major contributions. No woman has won a Fields Medal. The average Erdős number of Fields Medalists is 3.21, with a standard deviation of 0.87 and a median of 3.[7]

Fields Medalists 2010

Fields Medal – Elon Lindenstrauss

Elon Lindenstrauss is being awarded the 2010 Fields Medal for his results on measure rigidity in ergodic theory, and their applications to number theory.

Fields Medal - Elon Lindenstrauss, Hebrew University and Princeton University

Lindenstrauss has made far-reaching advances in ergodic theory, the study of measure preserving transformations. His work on a conjecture of Furstenberg and Margulis concerning the measure rigidity of higher rank diagonal actions in homogeneous spaces has led to striking applications. Specifically, jointly with Einsiedler and Katok, he established the conjecture under a further hypothesis of positive entropy. It has impressive applications to the classical Littlewood Conjecture in the theory of diophantine approximation. Developing these as well other powerful ergodic theoretic and arithmetical ideas, Lindenstrauss resolved the arithmetic quantum unique ergodicity conjecture of Rudnick and Sarnak in the theory of modular forms. He and his collaborators have found many other unexpected applications of these ergodic theoretic techniques in problems in classical number theory. His work is exceptionally deep and its impact goes far beyond ergodic theory.

Fields Medal – Ngô Bảo Châu

Ngô Bảo Châu is being awarded the 2010 Fields Medal for for his proof of the Fundamental Lemma in the theory of automorphic forms through the introduction of new algebro-geometric methods.

Fields Medal - Ngô Bảo Châu,
Université Paris-Sud

In the 1960's and 70's Robert Langlands formulated various basic unifying principles and conjectures relating automorphic forms on different groups, Galois representations and L-functions. These led to what today is referred to as the Langlands programme. The main tool in establishing some cases of these conjectures is the trace formula and in applying it for the above purposes a central difficulty intervenes: to establish some natural identities in harmonic analysis on local groups as well as ones connected to arithmetic geometric objects. This problem became known as the Fundamental Lemma. After many advances by a number of researchers in 2004, Laumon and Ngô established the Fundamental Lemma for a special family of groups, and recently Ngô established the Lemma in general.

Ngô's brilliant proof of this important long standing conjecture is based in part on the introduction of novel geometric objects and techniques into this sophisticated analysis. His achievement, which lies at the crossroads between algebraic geometry, group theory and automorphic forms, is leading to many striking advances in the Langlands programme as well as the subjects linked with it.

Fields Medal – Stanislav Smirnov

Stanislav Smirnov is being awarded the 2010 Fields medal for the proof of conformal invariance of percolation and the planar Ising model in statistical physics.

Fields Medal - Stanislav Smirnov, Université de Genève

It was predicted in the 1990's, and used in many studies, that the scaling limit of various two dimensional models in statistical physics has an unexpected symmetry, namely it is conformally invariant. Smirnov was the first to prove this rigorously for two important cases, percolation on the triangular lattice and the planar Ising model. The proof is elegant and it is based on extremely insightful combinatorial arguments. Smirnov's work gave the solid foundation for important methods in statistical physics like Cardy's Formula, and provided an all-important missing step in the theory of Schramm-Loewner Evolution in the scaling limit of various processes.

Fields Medal – Cédric Villani

Cédric Villani is being awarded the 2010 Fields Medal for his proofs of nonlinear Landau damping and convergence to equilibrium for the Boltzmann equation.

Fields Medal – Cédric Villani, Institut Henri Poincaré

One of the fundamental and initially very controversial theories of classical physics is Boltzmann's kinetic theory of gases. Instead of tracking the individual motion of billions of individual atoms it studies the evolution of the probability that a particle occupies a certain position and has a certain velocity. The equilibrium probability distributions are well known for more than a hundred years, but to understand whether and how fast convergence to equilibrium occurs has been very difficult. Villani (in collaboration with Desvillettes) obtained the first result on the convergence rate for initial data not close to equilibrium. Later in joint work with his collaborator Mouhot he rigorously established the so-called non-linear Landau damping for the kinetic equations of plasma physics, settling a long-standing debate. He has been one of the pioneers in the applications of optimal transport theory to geometric and functional inequalities. He wrote a very timely and accurate book on mass transport.

Famous Mathematics Quotes

- Number theorists are like lotus-eaters - having tasted this food they can never give it up.
  Leopold Kronecker (1823-1891)

- We must regard classical mathematics as a combinatorial game played with symbols.
  John Von Neumann (1903-1957)

Real-Life Applications of Mathematics

Algebra

1. Computer Science
2. Cryptology (and the Protection of financial accounts with encrypted codes)
3. Scheduling tasks on processors in a heterogeneous multiprocessor computing network
4. Alteration of pattern pieces for precise seam alignment
5. Study of crystal symmetry in Chemistry (Group Theory)

Differential Equations (Ordinary and Partial) and Fourier Analysis

1. Most of Physics and Engineering (esp. Electrical and Mechanical)
2. Sound waves in air; linearized supersonic airflow
3. Crystal growth
4. Cryocooler modeling
5. Casting of materials
6. Materials science
7. Electromagnetics analysis for detection by radar
8. Material constitutive modeling and equation of state
9. Underwater acoustic signal processing
10. Predict the evolution of crystals growing in an industrial crystallizer
11. Reentry simulations for the Space Shuttle
12. Rocket launch trajectory analysis
13. Trajectory prescribed path control and optimal control problems
14. Motion of a space vehicle

15. Aircraft landing field length
16. Design and analysis of control systems for aircraft
17. Underwater acoustic signal processing
18. Nonlinear dynamics
19. Large scale shock wave physics code development
20. Material constitutive modeling and equation of state
21. Molecular and cellular mechanisms of toxicity
22. Transport and disposition of chemicals through the body
23. Modeling of airflow over airplane bodies
24. Photographic development (Eastman Kodak)
25. Waves in composite media
26. Immuno-assay chemistry for developing new blood tests
27. Radio interferometry
28. Free mesons in nuclear physics
29. Seismic wave propagation in the earth (earthquakes)
30. Heat transfer
31. Airflow over airplane bodies (aerodynamics)

Differential and Computational Geometry

1. Computer aided design of mechanical parts and assemblies
2. Terrain modeling
3. Molecular beam epotaxy modeling (computational geometry)
4. Color balance in a photographic system
5. Optics for design of a reflector
6. Cryptology
7. Airflow patterns in the respiratory tract

Probability and Statistics

1. Calculation of insurance risks and price of insurance
2. Analysis of statistical data taken by a census
3. Reliability and uncertainty of large scale physical simulations
4. Speech recognition
5. Signal processing
6. Computer network design
7. Tracking and searching for submarines
8. Estimation of ocean currents (geostatistics)
9. Paint stripping using lasers
10. Onset and progression of cancer and pre-malignant cells
11. Determining launch schedules to establish and maintain prescribed satellite constellations (also uses Monte Carlo methods)
12. Radar track initiation
13. Aircraft survivability and effectiveness
14. Color sample acceptance tolerance correlation and prediction
15. Determination of sample sizes for color acceptability evaluation (uses advanced statistical methods)
16. Underwater acoustic signal processing
17. Reliability analysis of complex systems
18. Radio interferometry

Numerical Analysis

1. Estimation of ocean currents
2. Modeling combustion flow in a coal power plant
3. Airflow patterns in the respiratory tract (and diff. eqs.)

4. Regional uptake of inhaled materials by respiratory tract
5. Transport and disposition of chemicals through the body (and ODEs + PDEs)
6. Molecular and cellular mechanisms of toxicity (and ODEs + PDEs)
7. Reentry simulations for the Space Shuttle
8. Trajectory prescribed path control and optimal control problems
9. Shuttle/tank separation
10. Scientific programming
11. Modeling of airflow over airplane bodies
12. Electromagnetics analysis for detection by radar
13. Design and analysis of control systems for aircraft
14. Electromagnetics
15. Large scale shock wave physics code development
16. Curve fitting of tabular data

Operations Research and Optimization

1. Network formulation of cut order planning problem
2. Shade sorting of colored samples to an acceptable tolerance by hierarchical clustering
3. Inventory control for factory parts
4. Search for and tracking of submarines
5. Motion of a space vehicle
6. Aircraft survivability and effectiveness
7. Interplanetary mission analysis
8. Radio interferometry
9. Scheduling tasks on processors in a heterogeneous multiprocessor computing network
10. Coordinate measuring machine (optimization error modeling)
11. Optics for design of a reflector
12. Materials science
13. Reliability and uncertainty of large scale physical simulations
14. Microwave measurements analysis

University of Northern British Columbia 3333 University Way Prince George BC Canada V2N

Simple Real World Application of Linear Algebra

E. Ulrychov´a

Example: 1

Three people denoted by P_1, P_2, P_3, intend to buy some rolls, buns, cakes and bread. Each of them needs these commodities in differing amounts and can buy them in two shops S_1, S_2. Which shop is the best for every person P_1, P_2, P_3 to pay as little as possible? The individual prices and desired quantities of the commodities are given in the following tables:

Demand Quantity Of Food Stuff				
	roll	bun	cake	bread
P_1	6	5	3	1
P_2	3	6	2	2
P_3	3	4	3	1

Prices in shops S_1 *and* S_2		
	S_1	S_2
roll	1.50	1.00
bun	2.00	2.50
cake	5.00	4.50
bread	16.00	17.00

For example, the amount spent by the person P_1 in the shop $S_1$is:

$$6 \cdot 1.50 + 5 \cdot 2 + 3 \cdot 5 + 1 \cdot 16 = 50$$

and in the shop S_2

$$6 \cdot 1 + 5 \cdot 2.50 + 3 \cdot 4.50 + 1 \cdot 17 = 49$$

For the other people similarly. These calculations can be written using a product of two matrices

$$P = \begin{pmatrix} 6 & 5 & 3 & 1 \\ 3 & 6 & 2 & 2 \\ 3 & 4 & 3 & 1 \end{pmatrix}$$

(the demand matrix) and

$$Q = \begin{pmatrix} 1.50 & 1 \\ 2 & 2.50 \\ 5 & 4.50 \\ 16 & 17 \end{pmatrix}$$

(the price matrix). For example, the first row of the matrix

$$R = PQ = \begin{pmatrix} 50 & 49 \\ 58.50 & 61 \\ 43.50 & 43.50 \end{pmatrix}$$

expresses the amount spent by the person P_1 in the shop S_1 (the element r_{11}) and in the shop S_2 (the element r_{12}). Hence, it is optimal for the person P_1 to buy in the shop S_1, for the person P_2 in S_1 and the person P_3 will pay the same price in S_1 as in S_2

Example: 2

Three people (denoted by P_1, P_2, P_3) organized in a simple closed society produce three commodities Z_1, Z_2, Z_3. Each person sells and buys from each other. All their products are consumed by them, no other commodities enter the system (the "closed model"). The proportions of the products consumed by each of P_1, P_2, P_3 are given in the following table:

	Z_1	Z_2	Z_3
P_1	0.6	0.2	0.3
P_2	0.1	0.7	0.2
P_3	0.3	0.1	0.5

For example, the first column lists that 60% of the produced commodity Z_1 are consumed by P_1, 10% by P_2 and 30% by P_3. Thus, it is obvious that the sum of elements in each column is equal to 1.

Let us denote x_1, x_2, x_3 the income of the persons P_1, P_2, P_3. Then the amount spent by P_1 on Z_1, Z_2, Z_3 is $0.6x_1, +0.2x_2 + 0.3x_3$ the assumption that the consumption of each person equals his income leads to the equation $0.6x_1, +0.2x_2 + 0.3x_3 = x_1,$ similarly for the other persons. We obtain the system of linear equations:

$$0.6x_1, +0.2x_2 + 0.3x_3 = x_1$$

$$0.1x_1, +0.7x_2 + 0.2x_3 = x_2$$

$$0.3x_1, +0.1x_2 + 0.5x_3 = x_3$$

The system can be rewritten as the equation $Ax = x$ where

$$A = \begin{pmatrix} 0.6 & 0.2 & 0.3 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.1 & 0.5 \end{pmatrix} \quad and \quad X = (x_1, x_2, x_3)^r$$

Moreover, we assume the income to be nonnegative i.e. $x_i \geq 0 \ for\ i = 1,2,3$ (we denote it $x \geq 0$). We can rewrite this equation into the equivalent form $(A - I)x = 0$

$$\left(\begin{array}{ccc|c} -0.4 & 0.2 & 0.3 & 0 \\ 0.1 & -0.3 & 0.2 & 0 \\ 0.3 & 0.1 & -0.5 & 0 \end{array}\right)$$

An arbitrary solution of the system has the form $x = t(13,11,10)^T$ and it is $x \geq 0 \ for\ t \geq 0$

Thus to ensure that this society survives, the persons P_1, P_2, P_3 have to have their incomes in the proportions 13:11:10

Courtesy: WDS'06 Proceedings of Contributed Papers, Part I, 31–34, 2006

Me and My Father

When I was 4 yrs. old: My Father is The Best

When I was 6 yrs. old: My Father seems to know everyone

When I was 10 yrs. old: My Father is excellent but he is short tempered

When I was 12 yrs. old: My Father was nice when I was little

When I was 14 yrs. old: My Father started being too sensitive

When I was 16 yrs. old: My Father can't keep up with modern time

When I was 18 yrs. old: My Father is getting less tolerant as the days pass by

When I was 20 yrs. old: It is too hard to forgive my Father, how could my Mum stand him all the years

When I was 25 yrs. old: My Father seems to be objecting to everything

When I was 30 yrs. old: It's very difficult to be in agreement with my Father, I wonder if my Grand Father was troubled by my Father when he was a youth

When I was 40 yrs. old: My Father brought me up with a lot of discipline, I must do the same

When I was 45 yrs. old: I am puzzled, how did my Father manage to raise all of us

When I was 50 yrs. old: It's rather difficult to control my kids, how much did my father suffer for the sake of upbringing and protecting us

When I was 55 yrs. old: My Father was far looking and had wide plans for us, he was gentle and outstanding

When I became 60 yrs. old: My Father is The Best

Note that it took 56 yrs. to complete the cycle and return to the starting point **"My Father is The Best"**

Let's be good to our parents before it's too late and pray to **Allah** that our own children will treat us even better than the way we treated our parents.

NATIONAL ELIGIBILITY TEST (NET)

Joint CSIR-UGC Test for Junior Research Fellowship and Eligibility for Lectureship

Tentatively on 21st June, 2015

Date of Start of Line Submission: 13.02.2015

Date of close of Deposit of Fee (All Station): 11.03.2015

Date of close of Online Submission of Application (All Station): 12.03.2015

Important Links

- https://www.khanacademy.org/
- http://www.mathtv.com/
- http://www.hegartymaths.com/
- http://www.mymaths.co.uk/
- http://mathforum.org/
- http://www.cut-the-knot.org/
- http://www.sosmath.com/
- http://www.mathsnet.net/
- http://www.mavisresources.com/
- http://mathforum.org/library/view/4819.html

Professional Organizations

- American Mathematical Society (E-Math)
- Mathematical Association of America (MAA)
- European Mathematical Society (EMS)
- Canadian Mathematical Society (CMS)

Online Journals

- Electronic Journal of Combinatorics
- Electronic Journal Differential Equations (EJDE)
- Electronic Journal of Linear Algebra
- Electronic Transactions on Numerical Analysis
- INTEGERS: Electronic Journal of Combinatorical Number Theory
- New York Journal of Mathematics
- Journal of Statistics Education

Research

- Mathematical Sciences Research Institute (MSRI)
- Mathematics Archives
- Los Alamos National Laboratory Center for Nonlinear Studies (CNLS)
- National Academy of Sciences
- Guide to Available Mathematical Software
- Math in daily life

Finding Mathematics On The Web

- MathSciNet (References,etc)
- Search for Mathematics on the Internet
- Mathematics Directory
- Mathematical Atlas
- American Mathematical Society

Administration

- Health and safety briefing (Powerpoint, 550 KB)
- Computer Accounts
- Thesis Preparation and Submission

General

- LaTeX: Everything you Wanted to Know
- Statistics Encyclopedia
- Statistics Glossary
- Abbreviations of Names of Journals [PDF]

Jobs

- School of Mathematics and Statistics Careers Page
- Applied Mathematics Careers
- Searching for Employment in the Mathematical Sciences
- Worldwide Job Market for Mathematicians

National Centre for Mathematics

www.ncmath.org
(A joint centre of TIFR and IIT Bombay)
www.atmschools.org
Advanced Training in Mathematics Schools 2015
(Supported by National Board for Higher Mathematics)

Annual Foundation Schools			
Program	**Period**	**Venue**	**Organizers**
AFS-I	1st Dec - 29th Dec	IIT Delhi, Delhi	Rupam Barman
AFS-I	1st Dec - 29th Dec	Tezpur University, Assam	Nayandeep Deka Baruah Alok Goswami.
AFS-I	1st Dec - 29th Dec	Bhaskaracharya Pratishthan, Pune	V.V. Acharya S.A. Katre
AFS-II	4th May - 30th May	Shiv Nadar Univ, Gautam Buddha Nagar, UP	Amber Habib Rajendra Bhatia
AFS-II	11th May - 6th Jun	IISER, Trivandrum	K.N. Raghavan Sachindranath Jayaraman Viji Thomas
AFS-III	29th Jun - 25th Jul	HRI, Allahabad	Surya Ramana
Instructional Schools for Lecturers			
Linear Algebra	3rd May - 16th May	SGGS, Nanded	A.R. Patil
Functional Analysis	18th May - 30th May	IIT Kanpur	S. Chavan S. Dutta
Algebraic Topology	7th Dec - 19th Dec	Goa University	A.J. Jayanthan
Analysis and Differential Equations	10th Dec - 23th Dec	TIFR – CAM, Bangalore	Mythily Ramaswamy Venky Krishnan
Advanced Instructional Schools			
Naïve Set Theory & its applications	13th Apr - 2nd May	University of Kashmir	S.M.Srivastava Amin Sofi
Differential Topology	15th Jun - 4th Jul	NEHU Shillong	H.Mukherjee A.R. Shastri
Analytic Number Theory	1st Jun - 20th Jun	KIIT, Bhubaneswar	K. Srinivas
Algebraic Surfaces	4th May - 23rd May	IISER, Mohali	K. Paranjape
Commutative Algebra	14th Dec, 2015 1st Jan, 2016	CMI Chennai	Manoj Kummini A.V. Jayanthan
Riemannian Geometry	1st Dec – 20th Dec	IISER, Mohali	K. Gongopadhyay
Workshops			
Topology	16th Dec - 22nd Dec	RKMV University, Belur	Mahan Maharaj
Partial Differential Equations of Fractional order	6th Jul - 18thJul	TIFR, Bangalore	Imran Biswas K. Sandeep
Dynamics	3rd Aug - 8th Aug	IIT Bombay	S.G. Dani
Lie Groups	6th Apr - 11th Apr	IIT Bombay	M.S. Raghunathan

Mathematics Research Institutes in India

Chennai Mathematical Institute

Chennai Mathematical Institute is a centre of excellence for teaching and research in the mathematical sciences. It was founded in 1989 as a part of the SPIC Science Foundation, funded by the SPIC group in Chennai. Since 1996, it has been an autonomous institution.

The research groups in Mathematics and Computer Science at CMI are among the best known in the country. Recently, a research group has also been set up in Physics. The Institute has nurtured an impressive collection of PhD students.

The main areas of research in Mathematics pursued at the Institute are algebra, analysis, differential equations, geometry and topology. In Computer Science, the main areas of research are formal methods in the specification and verification of software systems, design and analysis of algorithms, computational complexity theory and computer security. In Physics, research is being carried out mainly in quantum field theory, mathematical physics and string theory.

Harish-Chandra Research Institute

The Harish-Chandra Research Institute (HRI) is an institution dedicated to research in mathematics, and in theoretical physics. It is located in Allahabad, India, and is funded by the Department of Atomic Energy, Government of India.

Research at HRI is focussed on Mathematics and Theoretical Physics. The academic community at HRI consists of faculty members, post-doctoral fellows, and graduate students.

The Institute has a graduate programme leading to the Ph.D. degree. Degrees for the programme are awarded by the HomiBhabha National Institute. Admissions to the graduate program take place through a Joint Entrance Screening Test, which is organized in collaboration with several other institutions, and an interview. Further, the Institute offers post-doctoral fellowships, and visiting positions at various levels.

Institute of Mathematical Sciences

The Institute of Mathematical Sciences (IMSc), founded in 1962 and based in the verdant surroundings of the CIT campus in Chennai, is a national institution which promotes fundamental research in frontier disciplines of the mathematical and physical sciences: Theoretical Computer Science, Mathematics, Theoretical Physics, as well as in many interdisciplinary fields. Around a hundred researchers, including faculty members, post-doctoral fellows and graduate students, are members of the Institute at any given time, in addition to a large number of visitors from all over the world.

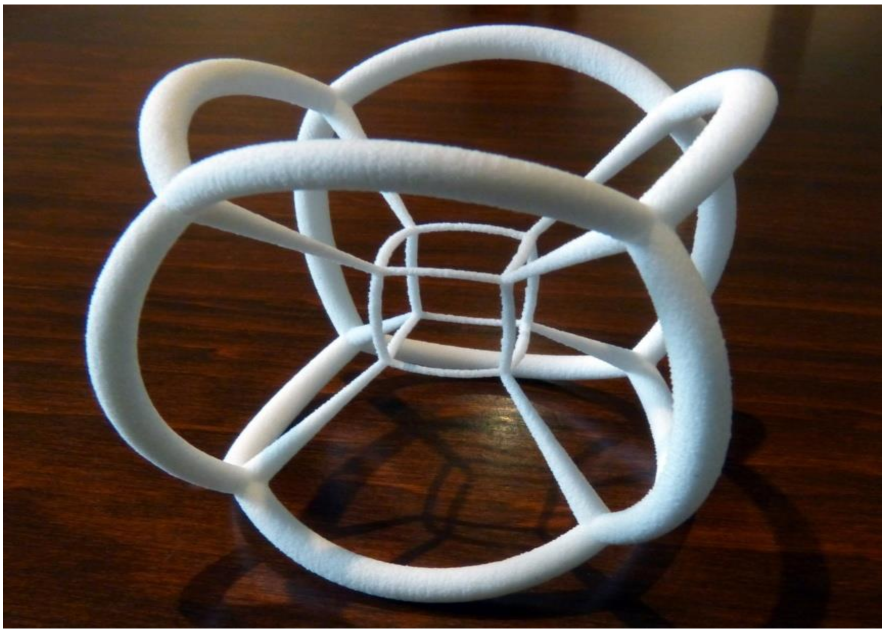
Tata Institute of Fundamental Research

The Tata Institute of Fundamental Research is a National Centre of the Government of India, under the umbrella of the Department of Atomic Energy, as well as a deemed University awarding degrees for master's and doctoral programs. At TIFR, we carry out basic research in physics, chemistry, biology, mathematics, computer science and science education.

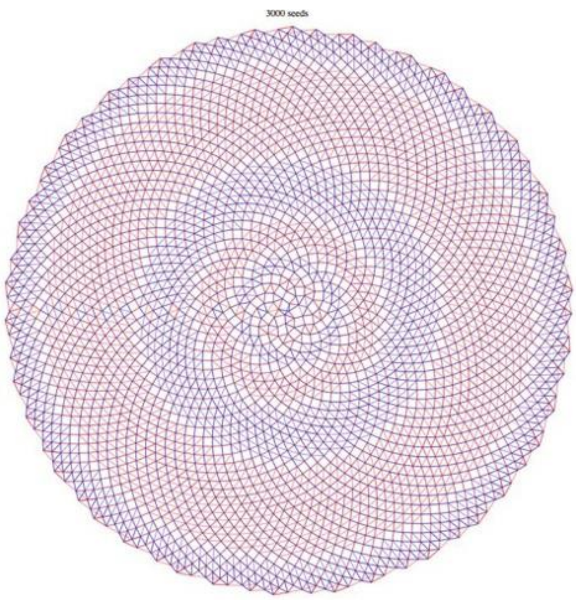
Kerala School of Mathematics

Kerala School of Mathematics (KSOM), an institution meant for advanced learning and research in Mathematics, is a joint venture of Kerala State Council for Science, Technology and Environment (KSCSTE), Government of Kerala and Department of Atomic Energy (DAE), Government of India. It is built in the traditional Kerala style architecture and is located in a picturesque setting surrounded by serene hillocks and lavish greenery. It is at a distance of about 15 km. from the seaside city Kozhikode in Malabar region of Kerala, India.

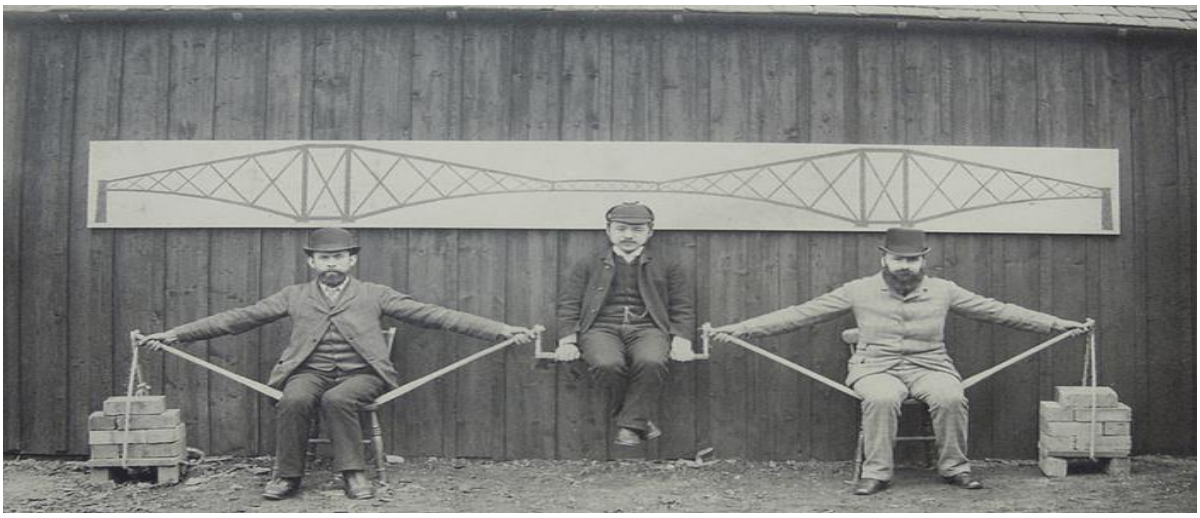
Mathematics Images



A 3D print of the tesseract, which is the four dimensional analogue of the cube. Sculpture by Saul Schleimer and Henry Segerman. Photograph: Henry Segerman



A representation of 3,000 seeds on a sunflower with spirals occuring a Fibonacci number of times: 1, 1, 2,3, 5, 8, ... Image: Ron Knott



A demonstration of the mathematical principles of the original Forth Bridge in Scotland performed at Imperial College in 1887. The central 'weight' is Kaichi Watanabe, one of the first Japanese engineers to study in the UK, while Sir John Fowler and Benjamin Baker provide the supports. Photograph: Imperial College

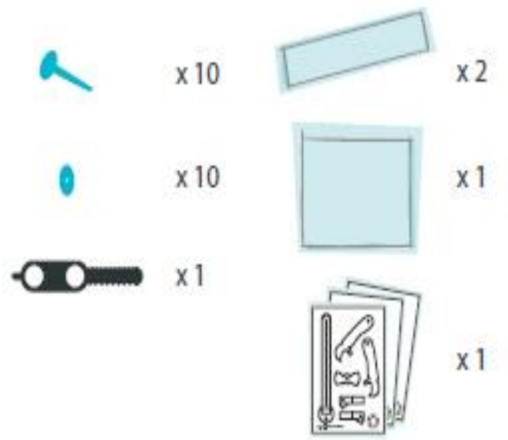
"Circles on Orthogonal Circles," by Anne Burns
(Long Island University, Brookville, NY)

"Circles on Orthogonal Circles," by Anne Burns (Long Island University, Brookville, NY)

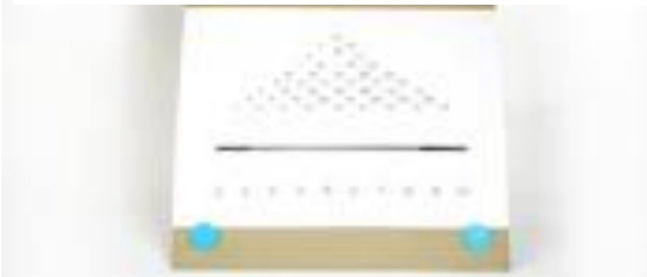
A loxodromic Möbius transformation has two fixed points, one attracting and the other repelling. Starting with a small circle around the repelling fixed point, and repeatedly applying the Möbius transformation, results in a family of circles that grow at first, each containing the previous one. Successive images eventually pass over the perpendicular bisector of the line connecting the fixed points and shrink as they are attracted to the other fixed point. Each circle in a second family of circles passes through the fixed points and is mapped to another circle in that family. Each circle in the second family is orthogonal to every circle in the first family.

Monkey Calculator

Does multiplication drive you bananas? With this clever calculating device you'll have the answers to every equation from 1x1 all the way through to 12x12 right at your fingertips. We created our Monkey Calculator out of recycled materials and Makedo parts available at *mymakedo.com*



To create the backing board for your calculator, you'll need one big rectangle that is 37cm x 28cm, and two 60 cm long strips.



Write the numbers onto your board exactly as they appear on the template. It's important that they're the same! Cut a slit in the board as shown on the template.

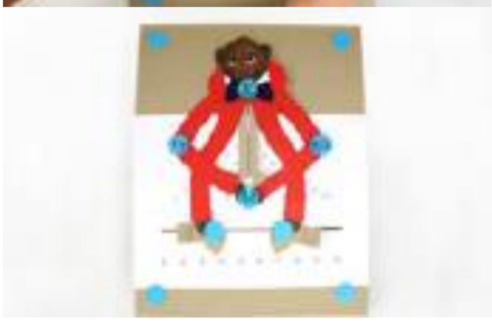
To make the board stand upright, fold the long strips at 5cm, 34 cm and 55 cm. Punch holes in the short tabs. Attach the longer end to the top of the board, and the shorter end to the bottom using Makedo Re-clips.

Cut out all the pieces of the template and decorate the Monkey. You can use bright colours to give personality, and don't forget to add a friendly smile.

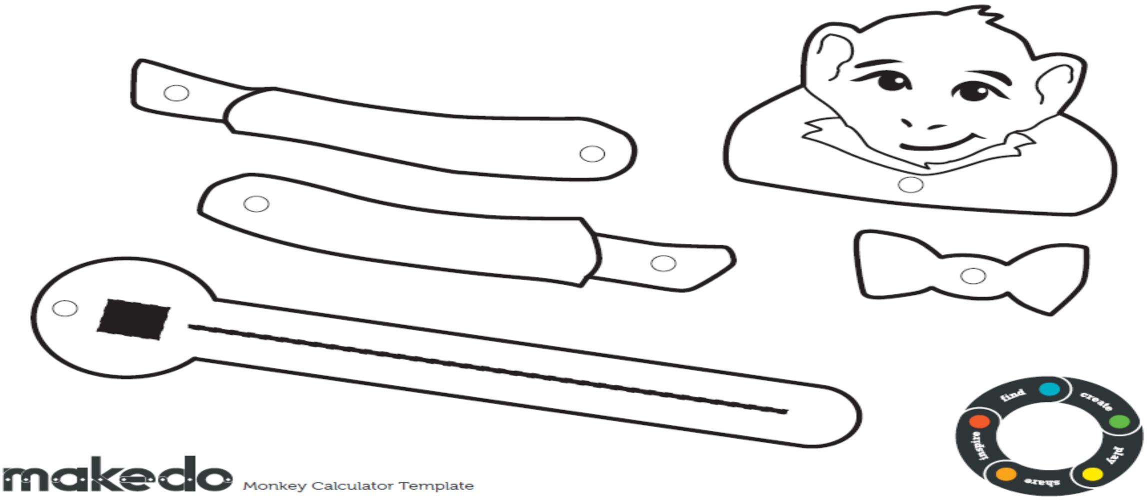
Using a Makedo Re-clip, attach in order: the bow tie, Monkey's head and top of Monkey's legs with the slider at the back, making sure all joins can move freely but aren't too loose at the same time.

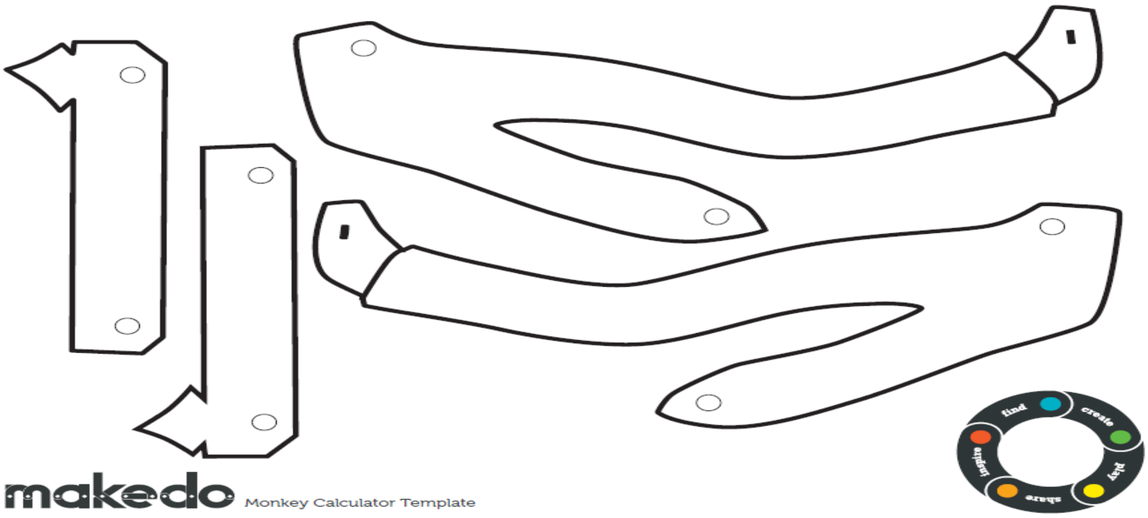
Connect the Monkey's elbows, then attach the hands to the bottom of the circle on the slider. Again, make sure that all joins can move freely.

Fold the arrows in half lengthways, bending the card to form handles. Attach the feet in front, sliding the Makedo Re-clip through the slit and secure behind the board as shown.



Your Monkey Calculator is complete! It's time to start counting. Remember to share what you've made with us at **mymakedo.com** , we'd love to see it!





10
9 20
18
8 30
7 16 27 40
6 14 24 36 50
5 12 21 32 45 60
4 10 18 28 40 54 70
3 8 15 24 35 48 63 80
2 6 12 20 30 42 56 72 90

1 2 3 4 5 6 7 8 9 10