FUNDAMENTALS OF PHYSICS

MECHANICS

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SEMESTER I: ALLIED – I - FUNDAMENTALS OF PHYSICS

UNIT – II : Mechanics:

Newton's law of gravitation – Kepler's laws of planetary motion – Gravitation constant G - Determination of G by Boy's method – Friction - laws of friction – Centre of gravity - Time period of compound pendulum - Centre of gravity of a solid hemisphere – meta center – meta centric height – ## Determination of the metacentric height of a ship ##.

Introduction to Mechanics

- Mechanics is a branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces.

- The study of mechanics involves many more subject areas. However, initial study is usually split into two areas; statics and dynamics.

- Statics is concerned with bodies that are either at rest or move with a constant speed in a fixed direction.

- Dynamics deals with the accelerated motion of bodies.

- Statics can be considered as a special case of dynamics where the acceleration is zero. In engineering, since many objects are designed with the intention that they are at rest or their motion remains constant, statics deserves special treatment.

Basic quantities :

- **Time** : Time is the measure of a succession of events and is a basic quantity in dynamics. Time is not involved in the analysis of statics problems. Time is a scalar quantity.
- **Length** : Length is needed to locate the position of a point in space and describes the size of a physical system. Once a standard unit of length has been defined, it is possible to define distances and geometric properties of a body as a multiple of the unit of length. Length is a scalar quantity.
- **Volume** : Volume is a measurement of the physical size of an object. It refers to how much space an object takes up. Volume is a scalar quantity.
- **Mass** : Mass is a different measurement of the size of an object. The mass, measured in kilograms, depends only on the amount of matter forming the body. Mass is scalar quantity
- **Density** : Density is related to mass and volume. It is defined as the mass per unit volume. This means that an object that has a large mass but a small volume will have a large density. Density is a scalar quantity

- **Speed** : Speed is a measure of how quickly a body is moving. It is defined as distance travelled per unit time. Speed is a scalar quantity.

- **Force** : Forces are influences on a body or system which, acting alone, would cause the motion of that body or system to change. A system or body at rest and then subjected to a force will start to move. To work with forces we need to know the magnitude (size), direction and the point of application of the force. Forces are vector quantities.

- **Displacement**: Displacement is a measure of distance in a particular direction. Displacement is a vector quantity. `

- **Velocity** : Velocity is the rate of change of displacement with respect to time. Velocity is a vector quantity.

- **Acceleration** : Acceleration is the rate of change of velocity with respect to time. Acceleration is a vector quantity.

- **Momentum** : Momentum is defined as the product of an object’s mass and its velocity. This is a very important quantity in mechanics. It arises in many problems particularly those involving collisions. Momentum is a vector quantity

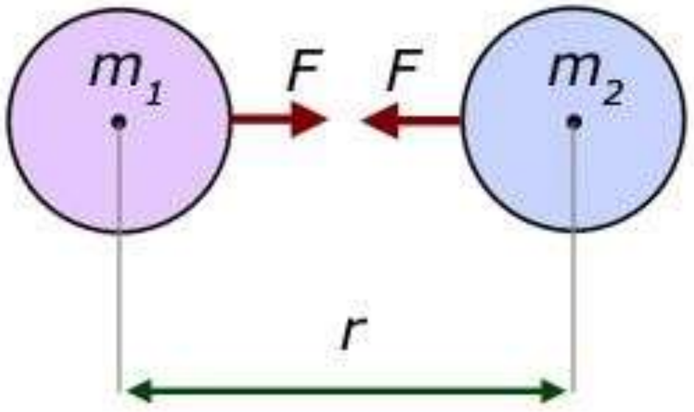
- Particles are bodies which can be treated as a point mass in a given context. For example, when modelling the motion of the planets around the Sun, the planets and Sun can be treated as particles. Much of basic mechanics study is concerned with objects that can be treated as particles.

- Connected particles arise in problems where two objects are attached in some way and both objects can be treated as particles. For example, two masses, connected by a string which passes over a pulley could be modelled as connected particles.

- Rigid bodies can be considered as combinations of particles in which all the particles remain at a fixed distance from one another both before and after applying a force i.e. there is no bending or stretching. For example a brick can in most circumstances be thought of as a rigid body. Many real life objects can be considered to be rigid bodies to a good approximation.

Newton's law of gravitation

- The universal law of gravitation states that every object in the universe attracts every other object with a force called the gravitational force. The force acting between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.



Derivation: For two objects of masses m1 and m2 and the distance between them r, the force (F) of attraction acting between them is given by the universal law of gravitation as:

$$F \,\alpha\, m_1 \text{ ------- (i)}$$

$$F \,\alpha\, m_2 \text{ ------- (ii)}$$

$$F \,\alpha\, \frac{1}{r^2} \text{ ------- (iii)}$$

From the above equations we can rewrite them as the following:

$$F \;\alpha\; \frac{m_1 m_2}{r^2} \text{ ------- (iv)}$$

If we remove the proportionality we get Proportionality constant **G** as the following:

$$F = G\frac{m_1 m_2}{r^2}$$

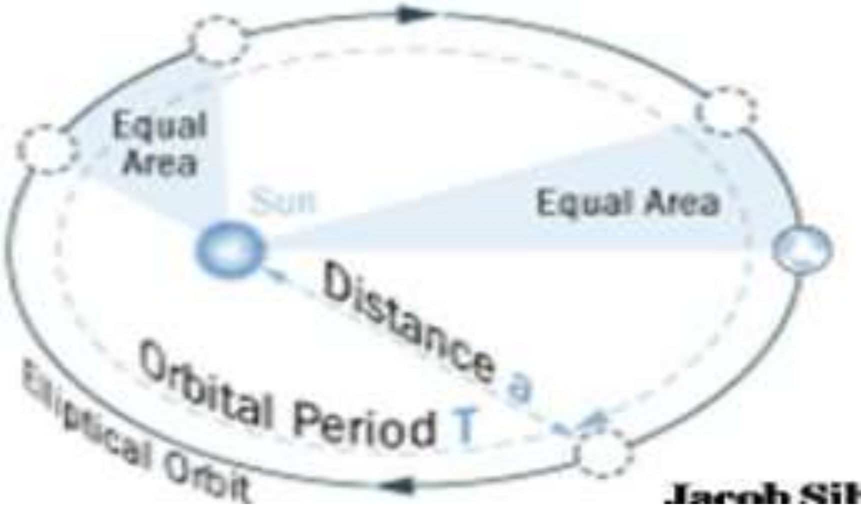
The above equation is the mathematical representation Of Newton's universal Law of gravitation

G is the Universal gravitational constant G, which was measured by Cavendish after more than 100 years G = 6.67 x 10 -11 N.m ^{2} /kg ^{2}

Kepler's laws of planetary motion

Kepler's first law: The orbital paths of the planets are elliptical, with the Sun at one focus.

Kepler's second law: An imaginary line connecting the Sun to any planet sweeps out equal areas of the ellipse in equal intervals of time.



Kepler's third law: The square of the planet's orbital period is proportional to the cube of its semimajor axis.

i.e. $T^2 = k\, a^3$ for some constant k

Boys' method for the determination of Gravitational Constant G

- The apparatus is shown diagrammatically in Figure 1, which consists of a fixed inner box X and rotated outer box Y
- In the inner draught proof box X, two gold spheres (a and b) 5 mm in diameter and with a mass of 3 g were suspended at different heights from either end of a bar C which was hung from a quartz fibre torsion wire
- In the outer box Y, the two large lead spheres (A and B), 115 mm in diameter and each with 7 kg were hung
- The spheres were mounted at different levels to minimise cross-attractive forces between a and B and between b and A. The whole apparatus had to be mounted on a stable base to prevent vibrations
- The plan view (Figure 2) shows the forces acting on the spheres. The forces between the small masses and the large masses cause the beam to twist through an angle θ as shown in the diagram
- From a lamp and scale arrangement is used to measure an angle θ

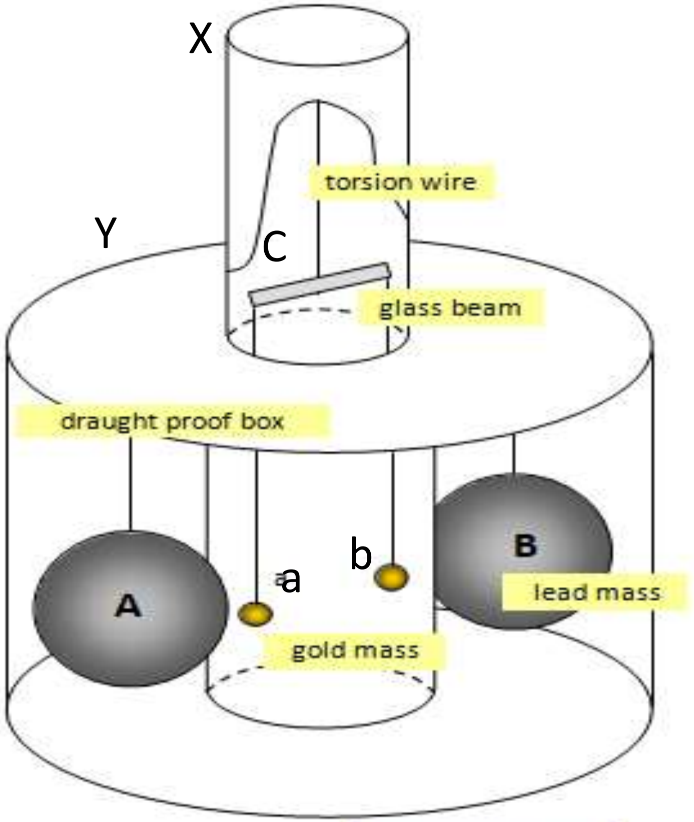


Figure 1

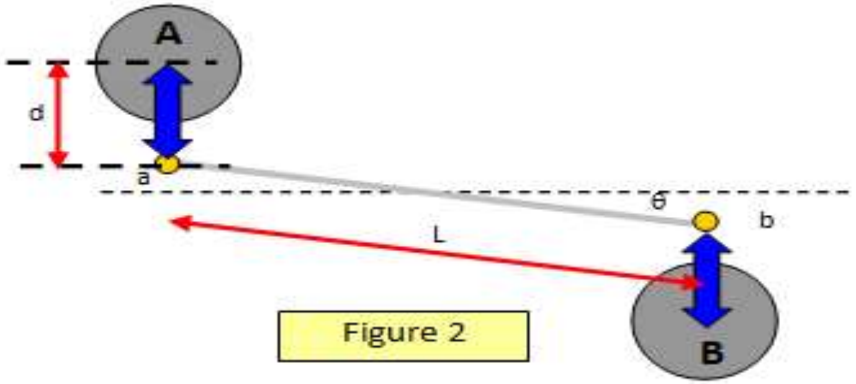


Figure 2

- Let the distance of the centres of **A to a** and **B to b** be **d**, and let the beam have a **length L**.

Then torque on the beam $\frac{GMmL}{d^2} = c\theta$

where **c** is the torque in the torsion wire per radian twist.

(Torque is the measure of the force that can cause an object to rotate about an axis. Hence, torque can be defined as the ***rotational equivalent of linear force***.)

- Therefore,

$$G = \frac{c\theta d^2}{mML}$$

- The torsional constant was determined by allowing the beam to oscillate and measuring the period of oscillation (T). Using the equation

$T = 2\pi\sqrt{I/C}$ where, $I = mr^2$

C can be found if the moment of inertia (I) of the beam and the small spheres is known.

The results obtained by him very accurate. The value obtained for G by Boys is $\mathbf{6.6576 \times 10^{-11}\ Nm^2\ kg^{-2}}$.

Advantages of Boys method

- The size of the apparatus is very much reduced
- By arranging the masses at different levels, the effect of the attraction of the heavier mass on the remote smaller mass is very much reduced
- By the lamp and scale arrangement, very small deflections can be measured accurately
- The use of a quartz fibre has made the apparatus very sensitive and accurate

Definition of G

- From Newton's law of gravitation

$$F = \frac{G\, m_1\, m_2}{r^2}$$

- If $m_1 = m_2 = 1$ kg and r = 1m, then F = G. Thus, "the Gravitational constant is equal to the force of attraction between two unit masses of matter unit distance apart".

L/T=(L/T)/T =L/T^2

Dimensions of G

Since, F = ma

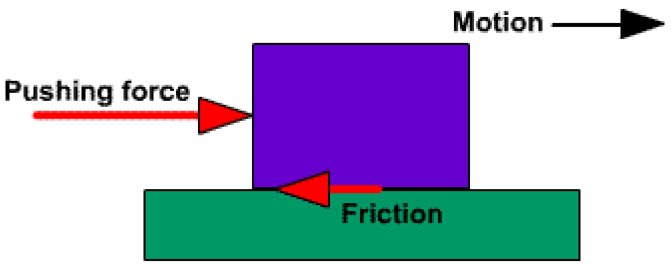
a = L/T^2 = ML/T^2

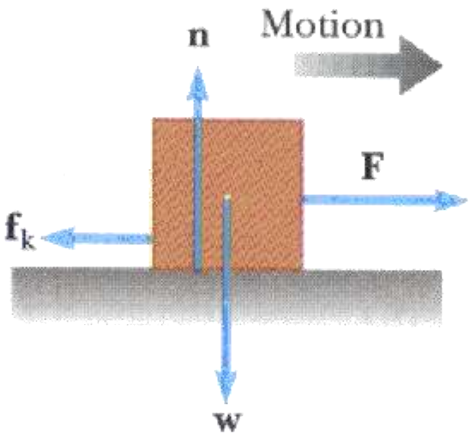
Dimensions of G are given by

$$G = \frac{Fr^2}{m_1\, m_2} \quad , \qquad G = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$$

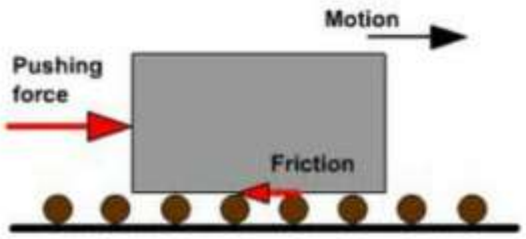
Friction

- Friction is the resistance to motion of one object moving relative to another.
- There are two main types of friction, static friction and kinetic friction.
- The frictional forces acting between surfaces at rest with respect to each other are called forces of static friction. The forces acting between surfaces in relative motion are called Kinetic or dynamic friction
- The maximum value of the frictional force between two bodies in contact is called limiting friction.
- The frictional force between two surfaces when one rolls over the other is called rolling friction
- In liquids, friction is the resistance between moving layers of a fluid, which is also known as viscosity. In general, more viscous fluids are thicker, so honey has more fluid friction than water.
- Friction plays an important part in many everyday processes. For instance, when two objects rub together, friction causes some of the energy of motion to be converted into heat. This is why rubbing two sticks together will eventually produce a fire.
- Friction is also responsible for the wear and tear on bike gears and other mechanical parts.





When an object rolls over other object like a wheel it is called as Rolling friction . it is easier to roll rather slide because rolling friction is lessser than sliding friction. Ex- Airport bags work on this principal only.



examples --- rolling friction- riding your bike or skateboard

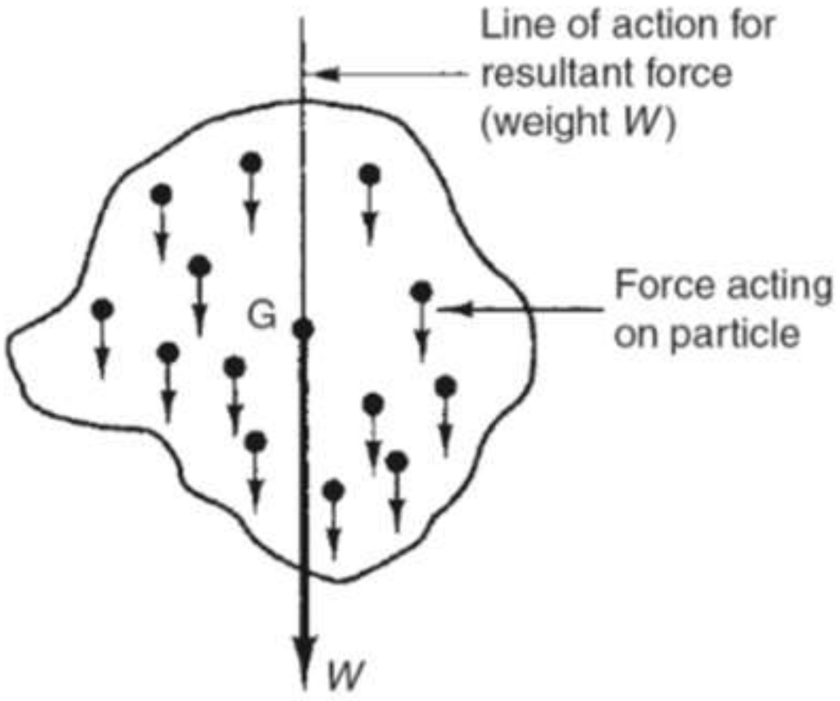
LAWS OF FRICTION

- When an object is moving, the friction is proportional and perpendicular to the normal force (N)
- Friction is independent of the area of contact so long as there is an area of contact
- The coefficient of static friction is slightly greater than the coefficient of kinetic friction.
- Within rather large limits, kinetic friction is independent of velocity
- Friction depends upon the nature of the surfaces in contact

Centre of Gravity

- Every body may be regarded as being made up of very large number of small particles.
- Each of these particles is being attracted towards the centre of the earth
- The force acting on these particles due to the earths attraction, can be regarded to be parallel to each other
- Now all these parallel forces can be replaced by a single resultant force (equal to weight of the body) passing through a fixed point G called the centre of gravity
- **"The centre of gravity of a body is that fixed point through which the resultant of the entire weight of the body acts in whatever position of the body"**
- The general formula for finding centre of gravity can be written as,

$$\bar{X} = \frac{\int x\, dm}{\int dm} \qquad \text{(OR)} \qquad \bar{Y} = \frac{\int y\, dm}{\int dm}$$



Time period of compound pendulum

- A simple pendulum theoretically has the mass of the bob concentrated at one point, but this is impossible to achieve exactly in practice. Most pendulums are compound, with an oscillating mass spread out over a definite volume of space.

- Let G be the centre of gravity of a compound pendulum of mass m that oscillates about a point O with OG = h If the pendulum is moved so that the line OG is displaced through an angle θ (Figure 1), the restoring couple is (torque):

$$- mgh\sin\theta = - mgh\theta \text{ ---------(1)} \qquad \text{if } \theta \text{ is very small.}$$

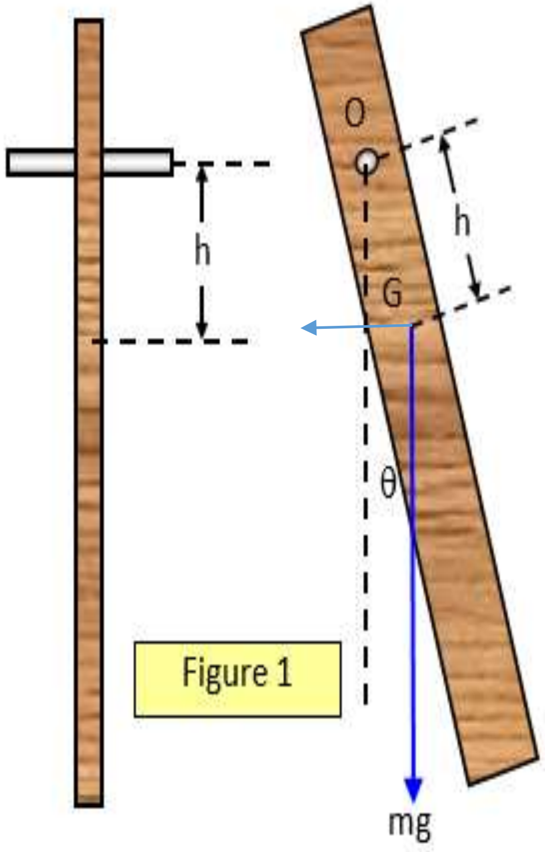
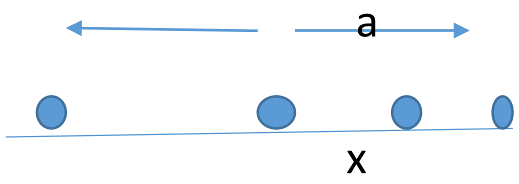


Figure 1

- Since torque act on pendulum is $\tau = I\alpha$ ---------(2) since, $F = ma$ in linear force where I is the moment of inertia about an axis through θ

- From Equ. 1 & 2 Therefore, $I\alpha = - mgh\theta$ ⇒ $\alpha = \frac{-mgh\theta}{I}$ $---(3)$

Therefore, angular acceleration is, $\alpha = -\omega^2 \theta$ ---------------(4)



$a = -\omega^2 x$

$\alpha = -\omega^2 \theta$

From Equ. 3 & 4 , $\omega = \sqrt{\frac{mgh}{I}}$ ------------(5)

Since , $T = \frac{2\pi}{\omega}$

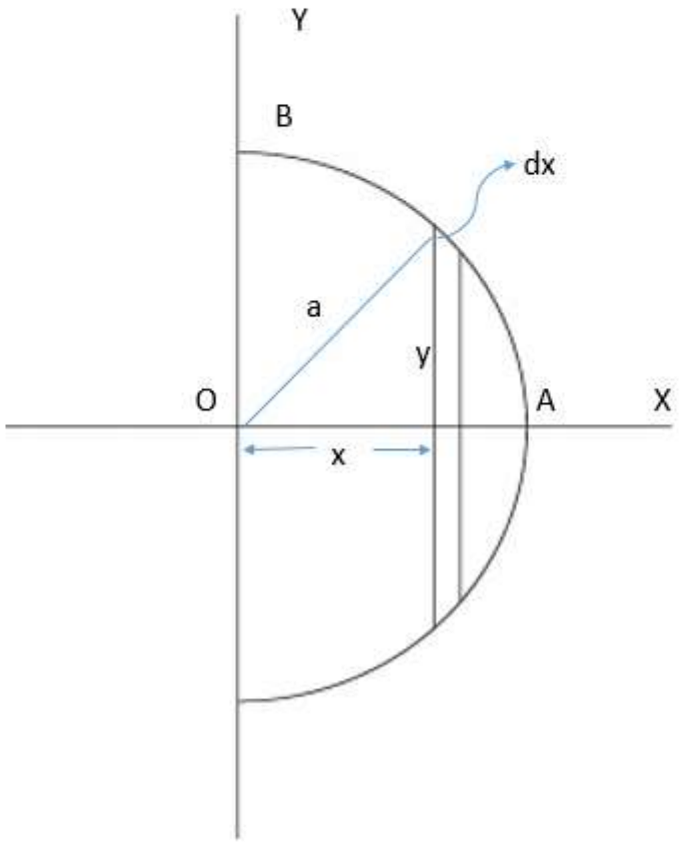
Hence, the periodic time is , $T = 2\pi\sqrt{\frac{I}{mgh}}$ --------------(6)

Centre of gravity of a solid hemisphere

- Let AB be the quadrant of the arc of a circle of radius ‘a’. The centre ‘O’ is taken as origin.
- If OA and OB are taken as axes, then the equation of the curve AB is,

$$X^2 + Y^2 = a^2 \quad \text{--------------------(1)}$$

- When the area AOB is rotated about the axis OA will get the volume of the hemisphere.
- Due to symmetry of the hemisphere, the centre of gravity lies on the axis OA. Hence $\overline{Y} = 0$.
- Consider a circular disc of thickness dx at a distance ‘x’ from ‘O’.
- Let Area of the disc is, $A = \pi y^2$ -----------------(2)
- Volume of the disc is, $V = \pi y^2\, dx$ -------------(3)
- If ρ is the density, then mass of this section is, $m = \pi y^2\, dx\, \rho$ ---------------(4)



❖ The centre of gravity of the circular disc will lie at (X,0), its centre. Therefore using the formula,

$$\bar{X} = \frac{\int x\, dm}{\int dm}$$

$$= \frac{\int_0^a x\, \pi y^2 dx\, \rho}{\int_0^a \pi y^2 dx\, \rho}$$

$$= \frac{\int_0^a x\, y^2 dx}{\int_0^a y^2 dx} \qquad \text{---------------(5)}$$

We know that, from equ. (1) $\quad Y^2 = a^2 - x^2$

$$\bar{X} = \frac{\int_0^a (a^2 - x^2) x\, dx}{\int_0^a (a^2 - x^2) dx}$$

$$= \frac{\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a}{\left[a^2x - \frac{x^3}{3}\right]_0^a} \qquad \int x^n\, dx = \frac{x^{n+1}}{n+1}$$

$$\bar{X} = \frac{\left[\frac{a^4}{2} - \frac{a^4}{4}\right]}{\left[a^3 - \frac{a^3}{3}\right]}$$

$$= \frac{\frac{a^4}{4}}{\frac{2a^3}{3}}$$

$$= \frac{a^4}{4} \times \frac{3}{2a^3}$$

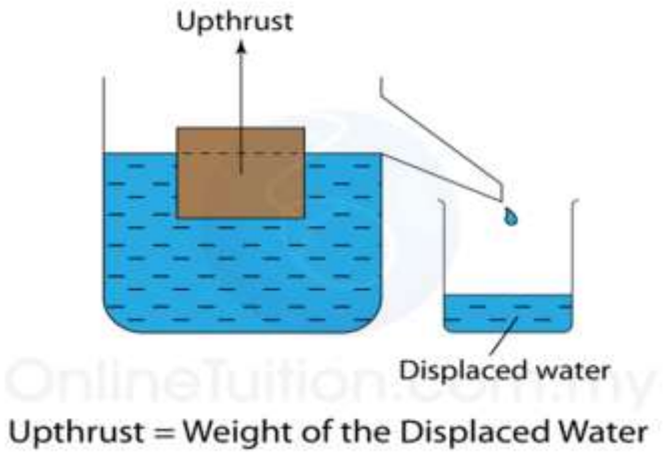
Therefore $\bar{X} = \frac{3a}{8}$

The centre of gravity of a solid hemisphere of radius 'a' lies on the axis at a distance $\frac{3a}{8}$ from the centre.

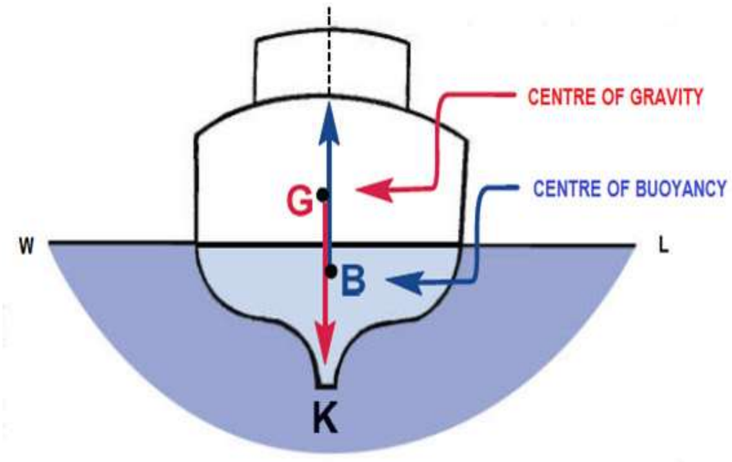
Laws of Floatation

❖ *A body floats in a liquid if:*

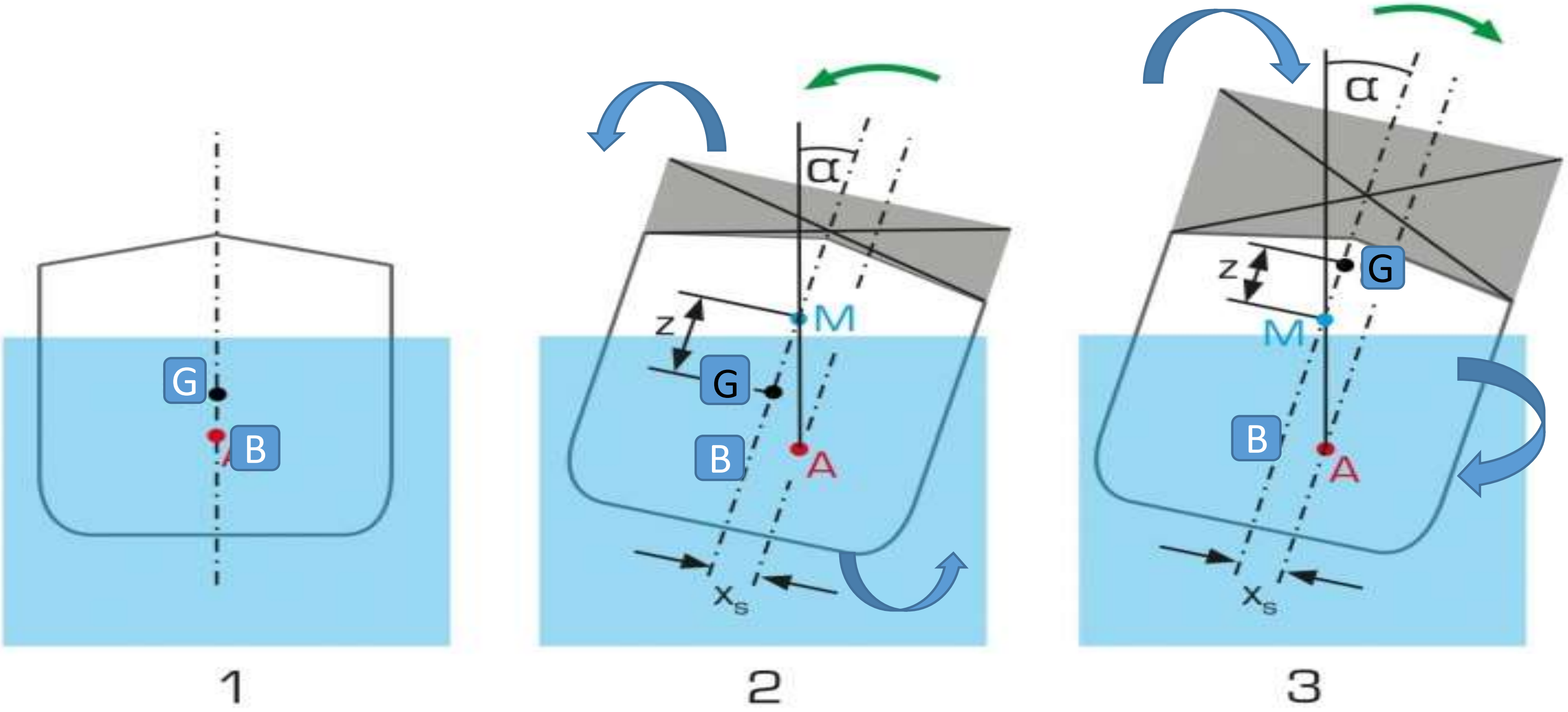
✓ When body floats in neutral equilibrium, the weight of the body is equal to the weight of liquid displaced by it.



✓ The centre of gravity of the body and centre of gravity of the displaced liquid (the centre of buoyancy) should be in one vertical line.



Stability of floating bodies



- ✓ The equilibrium of a freely floating body is said to be **stable**, if on being slightly displaced, the body returns to the original equilibrium position
- ✓ Here G is the CG of floating body and B is the centre of buoyancy. The line BG is vertical.

- ✓ When the floating body is slightly displaced, **A** is the new centre of buoyancy. The new centre of buoyancy **A** meets the original vertical line **BG** at point **M**, is called **Metacentre** of the floating body.

- ✓ **Meta centre:** When a floating body is slightly tilted from equilibrium position, the centre of buoyancy shifts. The point at which the vertical line passing through the new position of centre of buoyancy meets with the initial line is called **meta centre**.

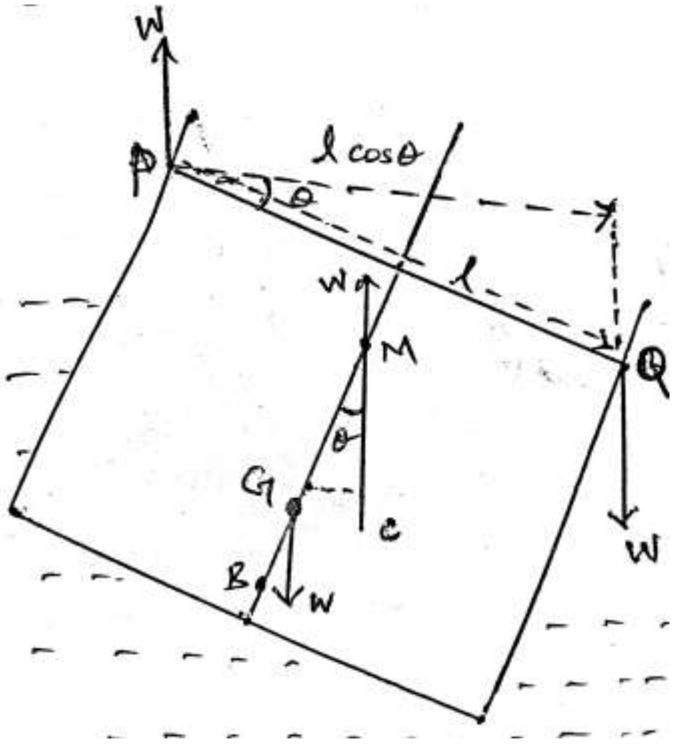
- ✓ The distance between **metacentre** and **centre of gravity** is called **Metacentric height (Z)**

- ✓ Since metacentre lies at **above the G**, the couple of forces due to **G** and **M** is **anticlockwise** and brings the floating body back to its original position. Hence, it is **stable equilibrium position**

- ✓ In further displacement of the floating body, the metacentre lies **at below the G**, the couple due to the forces at G and M is **clock-wise** and the couple tends to turn the body away from the equilibrium position. Hence this **equilibrium is unstable**.

- ✓ Hence for a floating body to be in stable equilibrium, **the metacentre must be always above the centre of Gravity of the body**

Experimental determination of the Metacentric height of a ship



- The weight of the ship ‘**W**’ is determined by the displacement method
- Two identical boats ‘**P**’ and ‘**Q**’ are attached a each side of the ship, with distance ‘**l**’ apart on the deck
- Filling the boats ‘P’ and ‘Q’ alternately with water is equivalent to moving a weight ‘**w**’ from ‘P to Q’ and ‘Q to P’ across the deck
- Turns the ship through an angle **θ**, which is determined by **plumb-line** suspended in the ship
- Now, this shift of weight ‘**w**’ from P to Q is equivalent to a downward force ‘w’ at Q and an upward force ‘w’ at P constituting a couple of moment is, **w l cos θ**
- If B and C be the original and altered positions of centres of buoyancy, G the centre of gravity of the ship and GM the metacentric height

- The weight W of the ship acting downwards at G and an equivalent upward thrust at the new centre of buoyancy C form a couple with an opposing moment is, **W x GM sin θ**

- Therefore, the equilibrium in the tilted position of ship,

$$W \times GM \sin\theta = w \times l \cos\theta$$

or $$GM = \frac{w\, l \cos\theta}{W \sin\theta}$$

$$GM = \frac{w\, l}{W \tan\theta}$$

$$\boldsymbol{GM = \frac{w\, l}{W\theta}}$$

Since θ being small, tan θ = θ

Thus knowing **W**, **w**, **l** and **θ**, we can easily calculated the metacentric height of a ship